Mathematics without apologies: portrait of a problematic vocation

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This book is an addition to the genre of insider-reports on the mathematical life written by distinguished mathematicians. It's much more than that, though, because Michael Harris is more than a mathematician; he is a Parisian intellectual. His suspicion of high-falutin' talk might cause him to resist that title, but he makes good jokes about Lacan, explores philosophical themes through literary analysis, treats popular culture as seriously as he does high culture, fills his book with references and allusions, argues by telling stories, reflects on the narrative conventions of those stories and is deeply suspicious of high-falutin' talk. If it walks like a duck and playfully explores the motives hidden beneath highly serious quacking like a duck, it's a duck. What must be maddening for the other Parisian intellectuals is that he does it rather well.

The title itself contains two allusions: to Hardy's *Mathematician's Apology* and to Weber's *Science as a Vocation*. This book resembles Weber's lecture on the values that animate scientists and the place of those values in the wider culture rather more than it resembles insider-reports such as Cédric Villani's *Birth of a Theorem*. Hardy's apology is a source of puzzles: why would one even try to justify mathematics to non-mathematicians? Why did Hardy go for the justification he chose? What other options are available? Harris observes that justifications of the material resources spent on pure mathematics usually refer to one or more of utility, truth and beauty. This, Harris claims, is a "Faustian" pact between mathematicians and their paymasters. "We promise Golden Geese, immutable truths, ineffable beauty. We collude in the misrepresentation of our values and our intentions." (p. xii). Routinely and publicly misrepresenting the deep motives of one's life's work results in alienation from oneself. This sense that there is something bogus about the apologies routinely offered for mathematics is the book's core problem and requires for its solution the Parisian activity of disclosure through juxtaposition and contextualisation. This may all come as an affront to those mathematicians who believed themselves to be perfectly sincere in offering Golden Geese, immutable truths and/or ineffable beauty to the non-mathematical world.

Harris appeals to Alasdair MacIntyre's distinction between the internal and external goods of a tradition. The external goods of a tradition are those that one might gain in other ways, such as money or fame. The internal goods are those that one can gain only by practicing the tradition and are usually difficult for non-practitioners to discern. Mathematicians, according to Harris, pursue an internal good, namely, pleasure "of an elusive, but nevertheless specific kind" (p. xi). Harris and Villani both reveal autobiographically that status-anxiety is also very important, but this does not undermine Harris's founding claim that there is something bogus about all that talk about utility, truth and beauty. Pure mathematics, he claims, is a 'relaxed field', which means that it has no strong motives outside itself. (As Villani's book makes vivid, the field may be relaxed but the mathematicians are not.) It is and must be free to develop according to its own internal lights and

imperatives. That is why the talk about utility, truth and beauty are bogus—these would be external motives, were mathematicians to take them seriously. Mathematicians perpetrate this talk because they normally pursue their researches at someone else's expense and you can't tell your funding agency that the point of the exercise is a rare kind of pleasure available only to the participants. MacIntyre's framework helps here because it supplies words to explain that the mathematician's pleasure is not just a subjective buzz, but rather an affective response to an objective good (the internal goods of a practice are as objectively real as goods of any other sort, however invisible they may be to non-practitioners). Mathematicians, we must believe, would not be interested in a pill that somehow gave them that elusive, specific mathematical pleasure without doing any mathematics.

This brings us to the sections in which Harris attempts to explain some number theory as if at a dinner party. The dinner guest on the receiving end is a performing artist. The fact that she has her own vocation with its own internal goods allows her to interrogate Harris's explaining number theorist about his tradition. They end their dialogue with a discussion of audience participation. The number theorist suggests, "Maybe the audience comes to watch you care about Nora [the protagonist of Ibsen's *A Doll's House*]". The performing artist retorts that "...the audience wants to care about Nora,... not just to [watch] some performer caring. Are you saying that the author [of this book about mathematics] expects readers to pick up the book just in order to watch mathematicians care about the Birch-Swinnerton-Dyer conjecture?" Faced with this challenge, the number theorist replies, "it's no small thing to be able to care about something like that" (p. 320). This is as much as he can say, because Harris also insists that solved problems and proven theorems do not offer the rare and specific pleasure for the sake of which mathematicians do mathematics. Nothing less than the current frontier will do, but that is only for professional practitioners.

The preface of this book starts with a quotation from Felix Hausdorff, or rather from his literary alter ego, Paul Mongré, on the subject of how difficult it is for a problematically self-aware 'modern' writer to write a preface. The final chapter briefly expounds Hausdorff's anti-metaphysical philosophy, with its Nietzschean worry that devotion to the truth might be bad for us, its claim that our reality only seems important to us because it is ours, and its insistence on the importance of the free play of thought. Harris uses an arch, jocular tone in much of the rest of the book to distance himself from the bogus pieties under discussion. That tone is eloquently absent from this chapter.