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# Grey System Theory Supported Markowitz Portfolio Optimization during High Volatility Periods[1]

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## Abstract

*Investment in stocks in high volatility periods is more difficult for investors relative to periods with lower volatility. In these high volatility periods, the efficiency levels of markets tend to decrease and the probability investors facing asymmetric information increase. As such, certain modern approaches are needed for not only stock selection, but also for portfolio creating with selected stocks. This study runs a Markowitz Portfolio Optimization supported by Grey Systems Theory on selected stocks from the BIST 30 Index for a time period during the 2008 Global Financial Crisis. The purpose of this study is to develop a model, which can be used for investor decision making in the periods of uncertainty. Additionally, this study also compares the performances of the developed model, Hybrid portfolios and Traditional portfolios. The results of the study show that the developed models with a modern approach can be applicable and successful for periods with high volatility.*

**Keywords: Decision Analysis; Portfolio Optimization; Financial Forecasting; Grey System Theory; GM(1,1) Model**

## 1. Introduction

Portfolio optimization, which implies the healthy allocation of financial resources among various assets, is an important problem in risk management.

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Expected return and risk are the most important parameters in portfolio optimization. Generally, investors aim to maximize returns while minimizing risk[1-5].

During times of financial crisis, when volatility in financial markets is relatively higher, the effective selection of stocks by investors is more difficult compared to periods of normal economic conjuncture. Therefore models reflecting current approaches are needed for both stock selection and portfolio formation. Among these contemporary approaches, Harry M. Markowitz's Mean-Variance (MV) model is often mentioned as the most effective optimization model[6-8]. Markowitz MV model is an optimization model that takes into consideration the expected return and risk of financial assets being analyzed for consideration for portfolio inclusion[2,9]. Therefore it can be said that the mathematical foundation of the Markowitz MV model rests on expected return and risk. Hence, the accurate estimation of expected return will result in successful portfolio optimization using Markowitz MV Model[2-5,10] as given in Equation 1, 2, and 3.

$$E(r_p)=\sum_{i=1}^{n} w_i E(r_i) \tag{1}$$

Here, $E(r_p)$ is the expected return of portfolio p, $w_i$ is the weight of the i$^{th}$ stock in portfolio p, $E(r_i)$ is the expected return of the i$^{th}$ stock.

$$\sigma_p^2=\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{2}$$

$$\sigma_p=\left[\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}\right]^{1/2} \tag{3}$$

Here, $\sigma_p^2$ is the variance of portfolio p, $\sigma_p$ is the standard deviation of portfolio p, $w_i$ is the weight of the i$^{th}$ stock in portfolio p, $w_j$ the weight of the j$^{th}$ stock, $\sigma_{ij}$ is the covariance between the $i^{th}$ and $j^{th}$ stocks.

The operational fundamentals of the Markowitz MV model is primarily based on calculating, the expected return of financial instruments being considered for portfolio inclusion. Afterwards the probability of deviation for the expected return of the instrument is determined, in other words its risk is calculated. In this manner, the expected return and risk of the investment alternatives are determined. Subsequently the model determines the expected return and risk of the portfolio by giving random weights to each possible investment instrument. In the final stage of the model an optimization process is carried out where appropriate instruments for portfolio inclusion and their weights are determined. Optimization is usually based on the expected return and risk of the portfolio. Before optimization, either return maximization or risk minimization must be selected as the objective function. After the optimization process is completed the optimum financial instrument combination or portfolio serving the objective function is obtained [11].

## 2. Literature Review

Various methods such as ARIMA models, regression analysis, artificial neural networks etc. can be used for estimating the expected return. But these methods require an adequate number of data points for effective estimation. For estimation problems where a limited number of data points are available GM(1,1) an estimation model based on the Grey System Theory (GST) has found acceptance and use in finance literature in the recent years. This stream of research includes a wide range of application of GM(1,1) models in finance context and include: Xia and Wong[12] developed a Grey Forecasting Model to improve accuracy of sales forecasting. Chen and Guo[13] where Grey Markov Model is used in forecasting firm specific financial crisis. Lin and Wu[14] analyzed banks' credit risk using Grey Relational Analysis. Wu and Chen[15] applied the GM(1,1) model for exchange rate estimation in post financial crisis periods. Askari and Askari[16], compared the performance of GM(1,1) and Improved GM(1,1) models to ARIMA models for forecasting London gold market prices. Hamzaçebi and Pekkaya[17] used Grey Relational Analysis for stock selection. Wu, Lin and Tsai[18] evaluated the business performance of wealth management banks in Taiwan using Analytical Hierarchy Process and Grey Relational Analysis. Fang-Min and Wang-Ching[19] developed a Grey Decision Model to categorize firms as "normal" and "abnormal" in developing a precautionary model to preserve investors and creditor firms against crises. Kayacan et al.[20] analyzed the performance of various Grey Models in predicting US/Euro parity. Huang and Jane[21] developed an automated stock prediction and portfolio selection mechanism by combining Autoregressive Exogenous Prediction Model with Grey Systems Theory and Rough Set Theory. Kung and Yu[22] predicted index futures returns in American, European and Asian financial market using GARCH/TGARCH, GM(1,1), GM(1,N), and GM(1,1) rolling models. Lim et al.[23] compared the performance of GM(1,1) Model and Backpropagation Artificial Neural Networks in predicting closing prices for online auctions. Li et al.[24] predicted the weekly closing values of Taiwanese stock index using a nonlinear GM(1,1) model they developed. Çukur et al.[25] employed Grey Prediction Models for predicting financial variables. Wen-han et al.[26] developed an early warning system for a financial crisis by predicting trend changes in credit volume in the Chinese financial industry using GM(1,1) model. Ho[27] employed Grey Relational Analysis (GRA) method for analyzing operational performance of three Taiwanese commercial banks. Yonghzong and Hongjuan[28] predicted short term and long term returns of Shanghai stock index using GM(1,1) and GM(1,1) Verhulst models. Qianyu[29] analyzed the financial marketization levels of six states in three different regions of China using Grey Target Approach. Chuang et al.[30] also employed GM(1,1) model for forecasting stock indexes. Wu and Chang[31] analyzed environmental cost allocation in Taiwan textile industry using Grey input-output analysis.

A Markowitz MV model based on the estimation output of a GM(1,1) is developed in the present research. This research can be said to target two different types of investors. Investors who are caught with stock investments in their portfolios comprise the first group. The second group is comprised of investors whose portfolios are composed of cash and want to take advantage of the high return potentials offered by high volatility. The primary objective of the first type of investors is assumed to be preservation of the principal. Earning positive returns for this type of investors is considered to be a secondary objective. The primary objective of the second type of investors is assumed to be earning high returns. The preservation of initial capital is assumed to be a secondary objective for these investors. The research is organized as follows: the third part explains the Grey System Theory, the fourth part undertakes an application using Borsa Istanbul

(Istanbul Stock Exchange) data and conclusion is the fifth part.

## 3. Grey System Theory

Grey System Theory (GST) was developed by the Chinese researcher Julong Deng in 1982. According to Deng, systems about which there is insufficient information are referred to as grey systems. In other words, grey refers to situations characterized by deficiency, incompleteness or uncertainty[32]. Deng states that for a system to be characterized as grey one of four states that causes incomplete information must exist[33]. These states are; (i) incomplete information about the parameters of a system, (ii) incomplete information about the structure of a system, (iii) incomplete information about the boundaries of a system, (iv) incomplete information about the behavior of variances in a system.

### 3.1 Grey Modelling

In GST, GM(h,N) refers to a grey model. In a GM(h,N) model the "GM" refers to Grey Model, the "h" in parentheses is the degree of the model and "N" is the number of variables in the model. Although there are various grey models used within the GST context, much of the research utilizing grey model prediction prefers to use GM(1,1) models since they provide efficient results and tend to be simpler to program. In real time forecasting the second most important issue, following performance, is the ease with which the model can be programmed[20]. Much of the extant empirical research posits the success and advantages of GM(1,1) models in terms of both performance and simplicity of programming.

### 3.2 GM(1,1) Prediction Model

GM(1,1) refers to a first degree grey model with a single variable. The GM(1,1) Model is used to discover relationships within time series, model according to these relationships and predict using this model. The differential equation of the GM(1,1) includes coefficients that are able to adjust according to instant changes. In other words, the GM(1,1) Model is able to make efficient predictions by adapting new pertinent data into the model. There is a precondition that the data to be used in the GM(1,1) Model needs to have a positive value and the time series have to have the same frequency, be it daily, weekly, monthly, etc.[20] Prediction through the GM(1,1) model requires three basic processes. These are, (i) Accumulated Generating Operation (AGO), (ii) Grey Modeling (GM) and (iii) Inverse Accumulative Generating Operation (IAGO)[34]. The GM(1,1) model performs these processes sequentially in order to model and predict a system[35]. GM (1,1) process steps can be expressed in the following equations[22,36]:

**Step 1:** Prediction using the GM(1,1) model begins with organization of the original data chronologically. In this way a time series is generated using the original data:

Assuming $X^{(0)}$ is an original time series;

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right) = \left(x^{(0)}(k); \quad k = 1,2,3,\ldots,n; \quad n \geq 4\right) \quad (4)$$

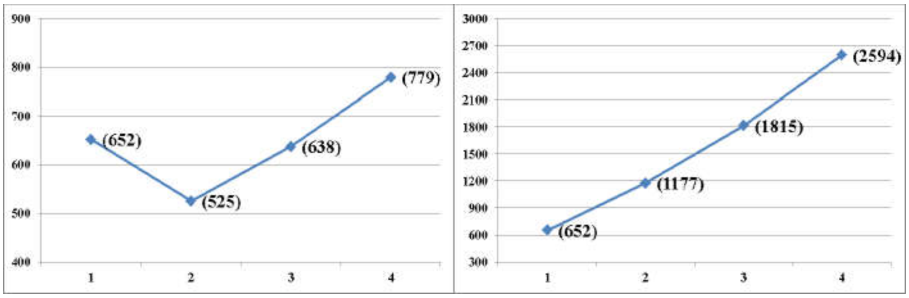
**Step 2:** After the initial step AGO is performed to reduce the randomness of the original series[37]. By applying AGO to the original series an accumulated series is obtained. The GM(1,1) Model transforms the original series into series with uniform increase. Through this transformation caused by AGO the randomness in the original series is effectively reduced. After this step, the first stage of the GM(1,1) Model is completed.

Here the AGO of $X^{(0)}$ series will be $X^{(1)}$. This can be stated through the following equation

$$X^{(1)} = \{x^1(1), x^1(2), x^1(3), ......, x^1(n)\} = (X^1(k); \quad k = 1,2,3,..., n; \quad n \geq 4) \tag{5}$$

$$X^{(1)}(k) = \left\{ \sum_{i=1}^{k} x^{(0)}(i) \qquad k = 1,2,3,..., n \right\} \tag{6}$$

In Graph 1, the line graphs for the original series and the generated accumulated series for a grey uncertainty problem are shown. For example, when $X^{(0)} = 652,525,638,779$ then $X^{(1)}$ happens $X^{(1)} = 652,1177,1815,2594$ .




***(a) original time series*** ***(b) accumulated time series***

**Figure 1** Line Graphs of the Original and Accumulated Time Series

Figure 1(a) is the graph for the original series while Figure 1(b) shows the new series obtained after AGO. As can be seen in Graph 1, while there was randomness in the original series the accumulated time series is uniform with almost exponential growth [38].

**Step 3:** Since the solution of first degree differential equations has an exponential form the new time series obtained from AGO is used to form the first degree differential equation. The obtained equation is used to predict the future behavior of the system [38]. As such, the first degree grey differential equation is formed from the accumulated time series as:

$$X^{(0)}(k) + aZ^{(1)}(k) = b, \quad k = 1,2,3,..., n \tag{7}$$

$$Z^{(1)} = \alpha X^{(1)}(k) + (1-\alpha) X^{(1)}(k-1), \qquad k = 1,2,3,..., n \tag{8}$$

where, $Z^{(1)}$ is the mean sequence of $X^{(1)}$ and $\alpha$ is equal to 0.5 generally [38,39].

**Step 4:** After the first degree differential equation of GM(1,1) mode is obtained the development coefficient a and direction coefficient b, which represents grey input are predicted. The most common method for prediction of a and b coefficients is Ordinary Least Squares (OLS). a and b coefficients are predicted through:

while,

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ .... & ... \\ -z^{(1)}(n) & 1 \end{bmatrix}, \qquad Y_N = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ ..... \\ x^{(1)}(n) \end{bmatrix} \qquad \text{and} \qquad A = \begin{bmatrix} a \\ b \end{bmatrix} \tag{9}$$

$$Y_N = BA \tag{10}$$

If the OLS equation is rewritten in matrix form the Equation 11 is obtained

$$A = (B^T B)^{-1} B^T Y_N = \begin{bmatrix} a \\ b \end{bmatrix} \tag{11}$$

**Step 5:** The first degree whitened differential equation or grey reflection equation as it is otherwise known is, obtained through the a and b coefficients estimated using OLS.

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{12}$$

**Step 6:** The estimated values of $X^{(1)}$, which is the accumulated time series are obtained using the first degree whitened grey differential equation.

$$\hat{x}^{(1)}(k+1) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{-(ak)} + \frac{b}{a}, \qquad k = 1,2,... \tag{13}$$

Following the solution of this equation the estimated values of the new series obtained through AGP will have been generated. This will complete the second, Grey Modelling, stage of the GM(1,1) model.

**Step 7:** The estimated values for the original series are obtained by performing IAGO on $X^{(1)}$ values.

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{14}$$

When the values for Equation 14 are put into Equation 13, Equation 15 is obtained. Then we solve Equation 15 and Equation 16 to obtain estimated values. When

$$\hat{x}^{(1)}(1) = x^{(0)}(1) \tag{15}$$

$$\hat{x}^{(0)}(k+1) = \left(1 - e^{a}\right)\left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-(ak)} \qquad k = 1,2,... \tag{16}$$

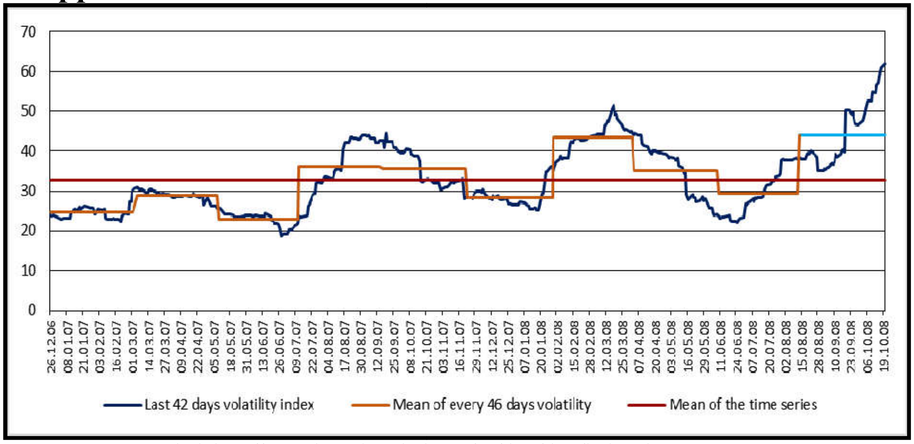
With this operation the third part of the GM(1,1) Model, IAGO is completed.

**Step 8:** After IAGO error analysis for grey predictions is performed.

$$e(k) = \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100 \tag{17}$$

If the predictions are found to be satisfactory after error analysis the process is completed. Otherwise the process is repeated from the AGO stage. This repetition is performed by assigning new k and beta values to the GM(1,1) Model. Repetitions are carried until satisfactory values are obtained.

## 4 .Application




**Source:** Borsa Istanbul, http://www.borsaistanbul.com/en/data/data/index-data, 21.08.2014.

**Figure** 2 42 Days Historical Volatility of BIST 30 Index

The data used in the model consists of BIST (Borsa Istanbul) 30 index daily closing values[40] for the period of 4th January 2007 - 20th October 2008. The data for this period represent a period exhibiting a large amount of volatility due to the effects of the global financial crisis. Figure 2, clearly illustrates the historical volatility index of BIST 30 Index for 42 days being examined in the present study. As can be observed from the graph the period under consideration is characterized by relatively high volatility. According to the Volatility Index[41] mean for the period between 26.12.2006-20.10.2008 is 32.90. Furthermore, index means for 46 day segments all the series exhibit significant variations. The period with highest mean is the period being forecasted and is shown in light blue in the graph. This period which covers 13.08.2008-20.10.2008 has a mean volatility of 44.11.

As can be seen in Graph 3 dramatic drops have taken in the place as a result of the crisis from the beginnings of 2008 on. Although the index has risen overall during this period in terms of trading days losses have been both more frequent and larger in magnitude. For potential investors considering stock investments in the BIST, using past period data for forecasting the future will not be very meaningful.

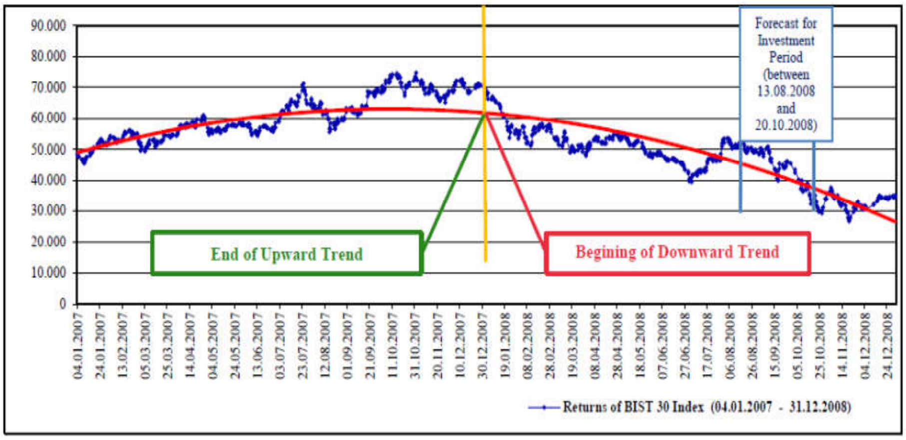



**Figure 3** Closing Values of BIST 30 Index (04.01.2007-31.12.2008)

The data for years prior to 2008 and data for post 2008 period begins to diverge, losing commonality. This is caused by the fact that while the trend prior to 2008 is a rising trend, while the trend from 2008 onwards is a falling trend. In other words, a new period with different characteristics begins in early 2008. Therefore, investors who want to invest in stocks and support their investment decisions with mathematical estimation techniques will face a problem. Using data for the period prior to 2008 may cause decreased estimation accuracy but the fact that many estimation techniques require past data will make it necessary to include 2007 and prior data for 2008 predictions. The solution to this problem rests on developing techniques that are able to model, predict and map the behavior of the current period with a minimal number of recent data. Therefore, GM(1,1) Model which has the ability forecast without requiring massive amount of data[32] was used to forecast stock returns and increase portfolio performance.

GM(1,1) Model based expected return estimations were utilized in 322 of the 327 portfolios formed in the present study. Two distinct approaches were used in the utilization of the model and this was followed by optimization and investment simulation. Investment simulation performance of the portfolios was analyzed by comparison to daily return of the BIST 30 index and returns obtained by alternative portfolios formed through established portfolio diversification approaches. The primary reason for employing three different techniques in the study was the desire to compare the models' performance and to a certain extent, obtain findings about portfolio optimization during periods of financial crises.

**4.1 Data Set**

The application consists of three distinct portfolio optimization applications using 25 stocks from BIST 30 Index. A total of 327 portfolios were formed within the context of the three applications and 25 stocks from the BIST 30 index was determined to be appropriate for inclusion in the portfolios. The portfolios were formed for the duration of 46 trading days between 13th August 2008 and 20th October 2008. The main reason why only BIST 30 Index stocks were included in the application is the fact that these are the 30 best stocks traded in the Borsa Istanbul. Also, the expectation that during periods of financial crisis, characterized by high volatility, that investment in high quality stocks will reduce risk is a factor. It is also widely accepted that the stocks included in the BIST 30 Index are the first stocks to reflect new information in the market. Therefore investment in BIST 30 Index stocks is accepted to be a more realistic approach during times of increased information asymmetry.

The data set was obtained through the Euroline Data Terminal of DirectFN Financial Data and Technology Services INC, which is one of the distributors give data distribution license by BIST. The study included 25 stocks chosen from BIST 30 Index; AKBNK, AKENR, ARCLK, ASYAB, DOHOL, DYHOL, ENKAI, EREGL, GARAN, HALKB, IHLAS, ISCTR, KCHOL, KOZAA, KRDMD, PETKM, SAHOL, SISE, SNGYO, TCELL, THYAO, TKFEN, TUPRS, VAKBN, and YKBNK. Five stocks were excluded from the application because they were not appropriate for analysis. The GM(1,1) Model used for estimation uses data for the period between 2nd January 2008 and 11th August 2008, however, BIZIM, EKGYO, KOZAL and TTKOM stocks were not being traded in the BIST during that period and had to be excluded. Also, since more past data was required for predicting MGROS stocks was required and since this was contrary to the purpose of the study it too was excluded. Therefore the total number of stocks included for price prediction was reduced to 25. The fact that the stocks included in the analysis covers 25 different industrial sectors can be considered to be positive feature as far as Traditional Portfolio Diversification and Modern Portfolio Theory are concerned. The rationale underlying this proposition is based on the expectation that different sectors will be effected differently by the same risk elements. Also, the fact that distinct sectors inherently have different characteristics carries forth the probability that some risks will be sector specific. Therefore it can be said that inclusion of stocks from different industrial sectors will contribute positively to reducing non-systemic risk.

Another point worth mentioning, regarding the data set used in the study, is the method of analysis adopted in data selection. The data set for the study was generated according to technical analysis approach. Technical analysis approach supposes that past period series being analyzed reflects all information about the market. Therefore in technical analysis the sole condition to be fulfilled is that time series regarding variables needs to be derived from past period data. Therefore in order to perform an analysis other economic or financial indicators need to be included in the analysis[42].

The reason for adopting a technical analysis approach in the study lies in the fact that this approach is also adopted by models used in prediction and optimization processes. The GM(1,1) Model used for prediction is a time series model. The principle workings of this model is based on making predictions regarding future periods based on past period data of time series. This structure exhibits one to one overlap with technical analysis approach. Furthermore, the Markowitz MV Model also employs past values of time series for calculating "expected rate of return" and "risk ratio" variables, necessary for optimization process. Therefore, calculations relevant to optimization process also overlap with technical analysis approach. In other words, since the structures of the models used are compatible to technical analysis approach, by extension, the structure of the application is based on technical analysis. Therefore, the data sets were determined and formed according to this structure.

**4.2 Optimization Process**

Three distinct portfolio optimization applications were carried out in the study and a total of 327 portfolios were formed towards this end.

The first application is portfolio optimization with Grey System Theory Supported Markowitz MV Model. In this application the daily rate of returns of the stocks was estimated for 46 trading days between 13.08.2008 and 20.10.2008 using GM(1,1) Model and stocks for which a drop in price was predicted was not included in the next day's portfolio. This was a proactive attempt to include in the Markowitz MV model optimization those stocks for which an increase was predicted. Optimization was carried out on the selected stocks using Markowitz MV model and the ratio of portfolio composition was determined. In determining portfolio inclusion rates the Sharpe Ratio, as given in Equation 18 was used.

$$S_p = \frac{E(r_p) - R_f}{\sigma_p} \tag{18}$$

Here, $S_p$ is the Sharpe Ratio of portfolio $p$, $E(r_p)$ is the expected return of portfolio $p$, $R_f$ is risk free interest rate, $E(r_p) - R_f$ is the risk premium for portfolio $p$, $\sigma_p$ is the total risk of portfolio $p$.

The treasury bill rate from the data distribution service of Central Bank of Turkey was transformed into daily series using Effective Rate formula, this is also risk free interest rate, as given in Equation 19 was used.

$$R_f = \left[\left(1 + \frac{i_n}{N}\right)^{N*T}\right] - 1 \tag{19}$$

Here, $R_f$ is risk free interest rate, $i_n$ is nominal interest rate, $N$ is the number of periods in a year and $T$ is the number of years.

At this stage, portfolio compositions were designed to achieve an efficient portfolio by taking into consideration the Sharpe Ratio values of portfolios consisting of stocks for which an increase was predicted. Hence, there were two stage share elimination and dynamic portfolios where stocks could be eliminated or included daily depending on estimations about their performance. In this stage of the application 230 portfolios were formed based on constraints outlined on Table 1. These portfolios are called Grey System Supported Markowitz Portfolios (GSSMP) in the study.

The second application is based on hybrid portfolios consisting of even proportions of stocks expected to rise the next day as determined through GM(1,1)

**Table 1** Investment Constraint Groups used in Optimization****

| Portfolio | Constraint for Ratio of Inclusion in the Portfolio to be Zero or Positive | Budget Constraint | Maximum Investment Ratio Constraint | Minimum Number of Stocks Constraint |
|---|---|---|---|---|
| *GSSMP-1 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i \leq 0.10$ | $h_i \geq 10$ |
| GSSMP-2 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i \leq 0.20$ | $h_i \geq 5$ |
| GSSMP-3 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i \leq 0.25$ | $h_i \geq 4$ |
| GSSMP-4 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i \leq 0.30$ | $h_i \geq 4$ |
| GSSMP-5 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i \leq 1.00$ | $h_i \geq 1$ |
| **GHP-1 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | - | $h_i \geq 1$ |
| GHP-2 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | - | $h_i \geq 4$ |
| ***TDP-1 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i = 0.10$ | $h_i \geq 10$ |
| TDP-2 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i = 0.10$ | $h_i \geq 10$ |
| TDP-3 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i = 0.083$ | $h_i \geq 12$ |
| TDP-4 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i = 0.077$ | $h_i \geq 13$ |
| TDP-5 | $w_i \geq 0$ | $\sum_{i=1}^{N} w_i = 1$ | $w_i = 0.04$ | $h_i \geq 25$ |

**Source:** Crama Y. and M. Schyns (2003). Simulated annealing for complex portfolio selection problems. European Journal of Orerational Research. 150(3), 546-571.

* GSSMP: Denotes Grey System Supported Markowitz Portfolio Constraints Group.

** GHP: Denotes Grey Hybrid Portfolio Constraints Group

*** TDP: Denotes Traditional Diversified Portfolio Constraints Group.

**** Since the share compositions of GSSMP and GHP were revised according to daily trend estimations, in conditions where sufficient stocks to satisfy minimum share constraint and maximum investment ratio constraints did not exist the portfolio was left in a liquid state.

Model predictions of expected returns. In this study determining stocks to be included in portfolios by predicting the direction of price change is accepted as a modern approach. On the other hand, preferring equal proportion portfolio content instead of determining portfolio content using the Markowitz MV model is a traditional approach. This is why the portfolios formed for the second application are referred to as hybrid portfolios. Within the application these portfolios are called Grey Hybrid Portfolios (GHP). The portfolios in the second application are also modified daily. The application consists of 92 portfolios formed in accordance with investment constraints shown on Table 1[43].

The third application consists of portfolios formed according to Traditional Portfolio Diversification. Since the share composition of these portfolios remain constant for the 46 trading day investment period, 5 portfolios were formed in accordance to investment constraints illustrated on Table 1. These portfolios are called Traditional Diversified Portfolios (TDP) in the study.

MATLAB R2011b 7.13 was used for developing the GM(1,1) Models that were used in forming the GSSMPs and GHPs for the first and second applications while, Solver add-on for Excel 2007 was used for optimization.

### 4.3 Findings and Discussion

This study includes portfolio optimization in accordance with three distinct approaches. Table 2 exhibits comparative lowest, highest and end of period rates of return for three different portfolio groups. These findings indicate that the GSSMP-5 portfolio had the highest positive return and TDP-4 to be the portfolio with the lowest positive return during the investment period. During the investment period TDP-2 was found to have the highest negative return while, GHP-1 was found to have lowest negative return. At the end of the investment period the highest return, although negative, was earned by GHP-1. The lowest return at the end of the investment period was earned by TDP-4. This illustrates that according to three criteria portfolios formed in accordance to Traditional Portfolio Approach were the most unsuccessful.

**Table 2** Performance of the Portfolios

| Portfolio | Highest Return | Trading Day | Lowest Return | Trading Day | End Return |
|---|---|---|---|---|---|
| **GSSMP -1** | 22.05% | 33rd day | -13.46% | 45th day | -7.30% |
| **GSSMP -2** | 13.15% | 19th day | -43.74% | 45th day | -38.96% |
| **GSSMP -3** | 13.76% | 33rd day | -35.01% | 45th day | -29.14% |
| **GSSMP -4** | 15.66% | 32nd day | -33.04% | 45th day | -26.60% |
| **GSSMP -5** | **26.08%** | 32nd day | -34.10% | 45th day | -23.94% |
| **GHP-1** | 14.97% | 33rd day | **-10.57%** | 45th day | **-5.63%** |
| **GHP-2** | 11.59% | 33rd day | -11.92% | 45th day | -7.04% |
| **TDP-1** | 2.19% | 3rd day | -43.32% | 45th day | -41.50% |
| **TDP-2** | 2.27% | 3rd day | **-46.79%** | 45th day | -44.63% |
| **TDP-3** | 2.55% | 3rd day | -45.22% | 45th day | -42.78% |
| **TDP-4** | **1.74%** | 3rd day | -46.54% | 45th day | **-44.75%** |
| **TDP-5** | 2.13% | 3rd day | -45.87% | 45th day | -43.78% |

Findings of the analysis can be better observed through figures. A general evaluation of portfolios formed for 46 trading days using GSSMP's, as can be seen in Figure 4, shows that portfolios generally generated positive returns of up to 20% in the initial stages of the investment period. Similarly, it can be seen in Graph 5 that GHP rate of returns reached about 15%. It is also observable from the graphs that by the middle of the investment period portfolio performance of both GSSMPs and GHPs dropped due to increased volatility. This can be explained by inability of GM(1,1) Models to fully model increased volatility causing wrong inaccurate predictions. Therefore, portfolio optimizations based on these predictions reduced the performance of the application. When daily returns of investment portfolios are compared to BIST 30 Index's daily returns it can be seen that during the investment period the portfolios generally performed better than BIST 30 Index, but at the end of the investment period only BIST 30 Index had a positive return.

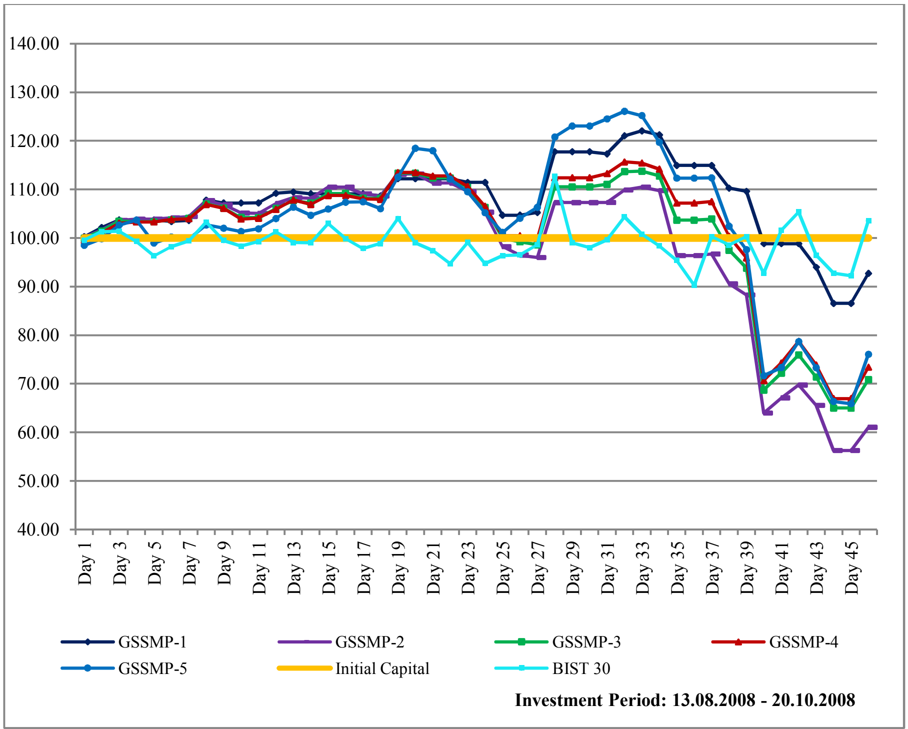



**Figure 4** 46 Days Return Performance of GSSMPs

Table 1 shows various constraint groups that were adhered to in forming the investment portfolios. Of the GSSMP's, GSSMP-5 had the highest return performance with 26% during the investment period, but the end of investment period return performance of this portfolio was -23.94%, a negative return. GSSMP-1, which had a return performance of 22% in the investment period had an end of investment period return of -7.3%.

The fact that GSSMP-5 had the highest rate of return in the investment period can be explained by the maximum share content constraint being set at "1". This constraint allows the portfolio to consist solely of a certain share. This liberal constraint allowed GSSMP-5 to include in greater proportions well chosen, high performing stocks. This same liberal constraint also allowed wrong share choices to cause greater losses in the portfolio, causing end of investment period return to be much lower than other portfolios with more stringent constraints, like GSSMP-1. During times of high volatility, like periods of financial crises, stock investment portfolios need to have constraints that ensure content diversity in order to minimize risk, as put forth by the present study. In more specific terms, based on the performance of GSSMP-1, we can suggest that during times of high volatility maximum single share content of a stock portfolio should have a maximum limit of 10%.

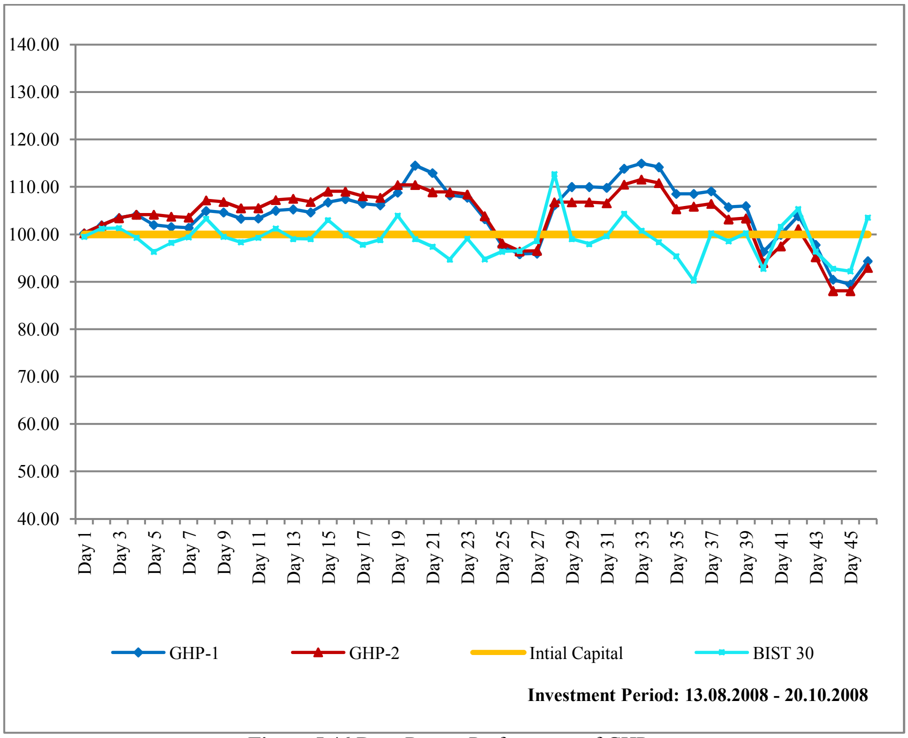



**Figure 5** 46 Days Return Performance of GHPs

When Figure 5 is analyzed a similar pattern can be observed for GHP-1 portfolios. The constraint group for GHP-1 portfolios is very similar to that of GSSMP-5 from the prior application with the exception of one constraint. On the other hand, using estimations on just the direction of change and risk/return optimization by Markowitz MV Model have prevented GHP-1 portfolio from having a very low end of investment period return. During the application period, the performance drop observed in return predictions was much lower for direction predictions. Therefore, GHP-1 portfolios which were formed using direction predictions instead of rate of return predictions had better end of investment period performance. Also the fact that GHP-1 portfolios were composed of equal proportions of included stocks enabled risk diversification. The fact is that GSSMP-1, GHP-1 and GHP-2 had values close to initial capital investment shows that these models are able to preserve initial capital.

These results show that during stages of financial crises where volatility has relatively stabilized there is a need to successfully predict rates of returns of stocks in addition to predicting direction of price trends. Results show that the models

applied are able to successfully predict direction of share price movements, but were not successful in predicting rates of return. Hence, it can be said that Markowitz MV model which places expected rate of return estimation in the center of optimization process is not able to effectively optimize portfolios.
The third portfolio type formed in the application consists of five portfolios formed in accordance to Traditional Portfolio Approach. Therefore, the stocks in these portfolios were randomly selected without a scientific basis. Two of these portfolios (TDP-1 and TDP-2) consisted of 10 determined by lottery from the 25 possible stocks. Two other portfolios (TDP-3 and TDP-4) included 12 and 13 stocks each, selected from the alphabetically ordered list of 25 stocks by skipping a share. The last portfolio, TDP-5, included all 25 stocks. All of the portfolios in this group had equal proportion of included stocks.

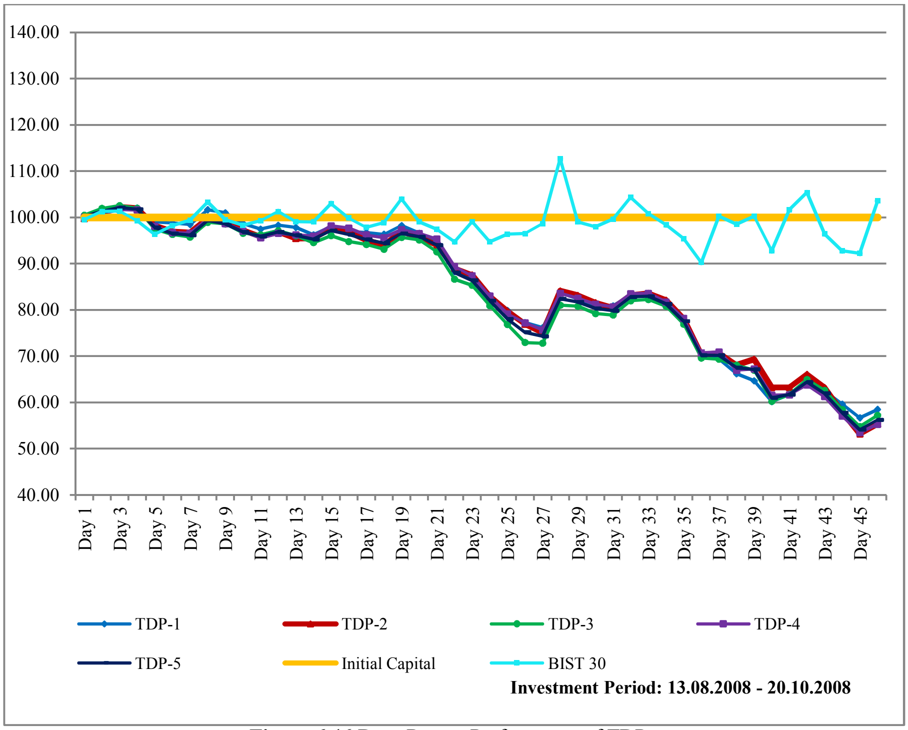



**Figure 6** 46 Days Return Performance of TDPs

As can be seen in Figure 6, 46 day investment period returns of TDPs are very different to GSSMP and GHP portfolios of the two other applications. TDPs had a negative rate of return beginning from the 5th day of the investment period and closed the investment period with negative returns. Only TDP-1 was able to generate positive returns in the 8th and 9th days, then generating negative returns until the end of the investment period like other TDPs. When we take into consideration that TDPs had a -40% to -45% rate of returns we can conclude that during times of crisis forming random portfolios without any mathematical prediction basis is not a very meaningful decision for investors.

It can be seen that price, direction predictions and optimization processes such as those performed on GSSMPs and price and direction predictions such as those done for TDPs can improve portfolio performance. Even though, the portfolios in the application closed the investment period with negative returns, they were able to perform much higher than BIST 30 Index during the investment period. This shows that mathematical based prediction and optimization techniques can contribute to investment decisions, help to preserve initial capital and obtain positive returns. This finding is in-line with extant literature.

## 5. Conclusion

The findings of this study support the need for high performance models capable of predicting the direction and price of stocks in high volatility periods to enable efficient optimization through Markowitz MV Model. The study shows that during high volatility periods “minimum share” constraint needs to be four, according to GHP findings, while a minimum of ten stocks are found through GSSMP findings.

The results show Grey Hybrid Portfolios to be especially useful for investors in periods of crisis. Also the applicability of the Markowitz MV Model in high volatility periods is observed to require the integration of a very powerful prediction model. This is due to the fact that expected return and risk parameters which form the basis of Markowitz MV Model needs to be obtained through estimation. The GM(1,1) Model which is used to estimate these parameters does have a high degree of accuracy for prediction during periods of high volatility like financial crisis, but it is also observed that it is not able to model extreme fluctuations in volatility.

In addition, due to the application period being a period of financial crisis, this study is stated to target two different investor groups. The first of these are investors who entered the crisis with portfolios composed of stocks. The second group is speculators who have liquid portfolios and desire to take advantage of high return potential offered by high volatility.

It is assumed that the primary aim of the first type of investors is to preserve their initial capital. Earning positive returns is considered as a secondary objective for these investors. The study has found that the first group of investors is able to realize their primary objectives through GHP-1, GHP-2 and GSSMP-1 portfolios. The investment constraints groups used in the formation of these portfolios which presented in Table 1 do serve as a guide for this type of investors. It is also found that the secondary objectives of this investor group can be partially achieved during the investment period, but since it is assumed that all investors completed the investment period the objective is not fully attained.

The primary objective of the second investor group is the same as the secondary objective of the first investor group. Therefore, within the investment period, provided that the expected rate of return is within the 15%-26% band, this objective can be attained. But since it is assumed that this investor group, like all other investors, completed the investment period, their primary objective is ultimately unattained. The secondary objective of the second group is assumed to be initial capital preservation. This objective of the second group is observed to be attained at the end of the period.

## References

[1] Sharpe, W.F.. Portfolio analysis. The Journal of Financial and Quantitative Analysis,1967, 2(2): 76-84.

[2] Michaud, R.O.. The Markowitz optimization enigma: Is 'optimized' optimal? Financial Analysts Journal, 1989, 45(1) : 31-42.

[3] Rubinstein, M.. Markowitz's "Portfolio Selection": A fifty-year retrospective. The Journal of Finance, 2002, 57(3) : 1041-1045.
[4] Gaoa, J., Xionga, Y. & Li, D.. Dynamic mean-risk portfolio selection with multiple risk measures in continuous-time. European Journal of Operational Research, 2015, Article in Press : 1–10.
[5] Vercher, E. & Bermúdez, J.D.. Portfolio optimization using a credibility mean-absolute semi-deviation model. Expert Systems with Applications,2015, 42:7121–7131.
[6] Chen, Y.-W., Poon, S.-H., Yang, J.-B., Xu, D.-L., Zhang, D., and Acomb, S.. Belief rule-based system for portfolio optimisation with nonlinear cash-flows and constraints. European Journal of Operational Research, 2012, 223 : 775–784.
[7] Deng, G.-F., Lin, W.-T., & Lo, C.-C.. Markowitz-based portfolio selection with cardinality constraints using improved particle swarm optimization. Expert Systems with Applications,2012, 39 : 4558–4566.
[8] Bekiros, S., Hernandez, J.A., Hammoudeh, S. & Nguyen, D.K.. Multivariate dependence risk and portfolio optimization: An application to mining stock portfolios. Resources Policy, 2015,46 : 1–11.
[9] Cornuejols, G., & Tütüncü, R.. Optimization methods in finance. New York: Cambridge University Press, 2007.
[10] Jorion, P.. Portfolio optimization in practice. Financial Analysts Journal,1992, 48(1), 68-74.
[11] Markowitz, H.M.. Portfolio selection. The Journal of Finance,1952, 7(1) : 77–91.
[12] Xia, M. and Wong, W.K. A seasonal discrete grey forecasting model for fashion retailing. Knowledge-Based Systems, 2014, 57:119–126.
[13] Chen, L.-H., & Guo, T.-Y.. Forecasting financial crises for an enterprise by using the Grey Markov forecasting model. Journal of Quality & Quantity, 2011, 45 : 911–922.
[14] Lin, S.-L., & Wu, S.-J.. Is grey relational analysis superior to the conventional techniques in predicting financial crisis? Expert Systems with Applications, 2011, 38 : 5119–5124.
[15] Wu, H., & Chen, F.. The application of Grey System Theory to exchange rate prediction in the post-crisis era. International Journal of Innovative Management, Information & Production,2011, 2(2) : 83–89.
[16] Askari, M., & Askari H. . Time series grey system prediction-based models: Gold price forecasting. Trends in Applied Sciences Research, 2011, 6(11) : 1287–1292.
[17] Hamzacebi, C., & Pekkaya, M.. Determining of stock investments with grey relational analysis. Expert Systems with Applications, 2011,38 : 9186–9195.
[18] Wu, C.-R., Lin, C.-T., and Tsai, P.-H. Evaluating business performance of wealth management banks. European Journal of Operational Research, 2010, 207 : 971–979.
[19] Fang-Min, L., & Wang-Ching, C.. A precaution diagnosis of financial distress via grey situation decision. The Journal of Grey System, 2010, 4 : 395–403.
[20] Kayacan, E., Ulutas, B., & Kaynak, O.. Grey system theory-based models in time series prediction. Expert Systems with Applications,2010, 37, 1784–1789.
[21] Huang, K.Y., & Jane, C.-J.. A hybrid model for stock market forecasting and portfolio selection based on ARX, grey system and RS theories. Expert Systems with Applications, ,2009,,,36 :5387–5392.
[22] Kung, L.-M., and Yu, S.-W.. Prediction of index futures returns and the analysis of financial spillovers—A comparison between GARCH and the grey theorem. European Journal of Operational Research, 2008, 186 : 1184–1200.
[23] Lim, D., Anthony, P., Mun, H.C. & Wai, N.K.. Assessing the accuracy of Grey System Theory against Artificial Neural Network in predicting online auction closing price. In proceedings of the International Multi Conference of Engineers and Computer Scientists, Vol 1, Hong Kong,2008.
[24] Li, G.-D., Yamaguchi, D., & Nagai, M.. The development of stock exchange simulation prediction modeling by a hybrid grey dynamic model. International Journal of Advanced Manufacturing Technology, 2008, 36 : 195–204.
[25] Çukur, S., Kotil, E., & Eryiğit, R.. Forecasting financial variables by the Grey Theory. ISE Review, 2007, 9(35) : 11–20.
[26] Wen-han, Y., Si-feng, L., & Yan, W.. The grey prediction of change trend of Chinese financial industry's credit size. The Journal of Grey System, 2006, 1 : 67–74.
[27] Ho, C.-T.. Measuring bank operations performance: An approach based on grey relation analysis. The Journal of the Operational Research Society, 2006, 57(4) : 337–349.
[28] Yongzhong, C., & Hongjuan, L.. A comparison of two grey models in predicting stock index. The Journal of Grey System, 2005,1, :73–76.
[29] Qianyu, Z.. Regional comparison of financial marketization degree via grey target approaching analysis. The Journal of Grey System, 2005, 3, 271 -276.
[30] Chuang, Y.-L., Hsu, M.-H., Wang, Y.-F., & Wang, Y.-H.. Forecasting stock price index using grey system. The Journal of Grey System, 2004, 2 :179–186.

[31] Wu, C.C., and Chang, N.B.. Grey input–output analysis and its application for environmental cost allocation. European Journal of Operational Research, 2003, 145 :175–201.
[32] Deng, J.. Introduction to Grey System Theory. The Journal of Grey System, 1989, 1(1):1-24.
[33] Lin, Y., Chen, M. & Liu, S.. Theory of grey systems: Capturing uncertainties of grey information. The International Journal of Systems and Cybernetics, 2004, 33(2) : 196–218.
[34] Mao, M., & Chirwa, E.C.. Application of grey model GM(1,1) to vehicle fatality risk estimation. Technological Forecasting & Social Change ,2006, 73(5) : 588–605.
[35] Lin, C.-B., Su, S.-F., & Hsu, Y.-T... High-precision forecast using grey models. International Journal of Systems Science, 2001, 32(5):610–611.
[36] Yao, A.W.L., & Chi, S.C.. Analysis and design of a Taguchi grey based electricity demand predictor for energy management systems. Energy Conversion and Management, 2004, 45 : 1205–1217.
[37] Hsu, C.-C., & Chen, C.-Y.. Applications of improved grey prediction model for power demand forecasting. Energy Conversion and Management, 2003, 44 :2241–2249.
[38] Zhou, P., Ang, B.W. & Poh, K.L.. A Trigonometric Grey Prediction Approach to Forecasting Electricity Demand. Energy, 2006, 31(14) : 2839–2847.
[39] Hamzacebi, H. & Es, H.A..Forecasting the annual electricity consumption of Turkey using an optimized grey model. Energy, 2014, 70 : 165–171.
[40] Borsa Istanbul (BIST), "BIST Indices", www.borsaistanbul.com, 2013.
[41] Borsa Istanbul, "Historical Volatility of BIST 30 Index", http://www.borsaistanbul.com/en/data/data/index-data, 2014.
[42] Murphy, J.J.. Technical analysis of the financial markets. USA: New York Institute of Finance,1999.
[43] Crama Y. and M. Schyns. Simulated annealing for complex portfolio selection problems. European Journal of Operational Research, 2003, 150(3) : 546-571.