Measuring the giant radio galaxy length distribution with the LoTSS DR2

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ABSTRACT

Context. Many massive galaxies launch jets from the accretion disk of their central black hole, but only ~103 instances are known in which the associated outflows form giant radio galaxies (GRGs, or giants); luminous structures of megaparsec extent that consist of atomic nuclei, relativistic electrons and magnetic fields. Large samples are imperative to understanding the enigmatic growth of GRGs, and recent systematic searches in homogeneous surveys constitute a promising development. For the first time, it is possible to perform meaningful precision statistics with GRG lengths, but a framework to do so is missing.

Aims. We measure the intrinsic GRG length distribution by combining a novel statistical framework with a LOFAR Two-metre Sky Survey (LoTSS) DR2 sample of freshly discovered giants. In turn, this allows us to answer an array of questions on giant radio galaxies. For example, How rare is a 5 Mpc giant compared to one of 1 Mpc? Given a projected length, how much larger is the intrinsic length expected to be? And what is the GRG number density in the Local Universe?

Methods. We assume the intrinsic GRG length distribution to be Pareto (i.e. of power-law form) with tail index ξ, and predict the observed distribution by modelling projection and selection effects. To infer ξ, we also systematically search the LoTSS DR2 for hitherto unknown GRGs and compile the largest catalogue of GRGs to date.

Results. We show that if intrinsic GRG lengths are Pareto distributed with index ξ, then projected GRG lengths are also Pareto distributed with index ξ. Selection effects induce curvature in the observed projected GRG length distribution: angular length selection flattens it towards the lower end, while surface brightness selection steepens it towards the higher end. We explicitly derive a GRG’s posterior over intrinsic lengths given its projected length, laying bare the ξ-dependence. We also discover 2050 GRGs within the LoTSS DR2, our sample more than doubles the known population. Spectacular discoveries include the largest, second-largest, and fourth-largest GRG known (l_p = 5.1 Mpc, l_p = 5.0 Mpc, and l_p = 4.8 Mpc), the largest GRG known hosted by a spiral galaxy (l_p = 2.5 Mpc), and the largest secure GRG known beyond redshift 1 (l_p = 3.9 Mpc). We increase the number of known GRGs whose angular length exceeds that of the Moon from 10 to 23; among the discoveries is the angularly largest known radio galaxy in the Northern Sky, which is also the angularly largest known GRG (ϕ = 2°). Combining theory and data, we determine that intrinsic GRG lengths are well described by a Pareto distribution, and measure the index ξ = −3.5 ± 0.5. This implies that, given its projected length, a GRG’s intrinsic length is expected to be just 15% larger. Finally, we determine the comoving number density of GRGs in the Local Universe to be n_GRG = 5 ± 2 (100 Mpc)^{-3}.

Conclusions. We develop a practical mathematical framework that elucidates the statistics of giant radio galaxy lengths. Through a LoTSS DR2 search, we also discover 2050 new giants. By combining both advances, we determine that intrinsic GRG lengths are well described by a Pareto distribution with index ξ = −3.5 ± 0.5, and that GRGs are truly rare in a cosmological sense: most clusters and filaments of the Cosmic Web are not currently home to a giant. Thus, our work yields new observational constraints for analytical models and simulations of radio galaxy growth.

Key words. Galaxies: active – jets – kinematics and dynamics – Radio continuum: galaxies

1. Introduction

When gas, dust and stars accrete onto a supermassive black hole (SMBH) in the centre of a galaxy, collimated jets arise along the Kerr rotation axis that blast some of the infalling matter into the intergalactic medium (IGM) (e.g. Blandford & Rees [1974]). In this process, the ejecta dissolve into a relativistic plasma that drags along a magnetic field and glows in synchrotron light. The resulting luminous structure is called a radio galaxy (RG); the central black hole that has generated it an active galactic nucleus (AGN).

It is increasingly clear that RGs and their AGN play an important role in galaxy evolution and cosmology. By heating the interstellar medium (ISM) or even expelling it from their host galaxies through galactic superwinds, AGN quench star formation (e.g. Di Matteo et al. [2005]). Beckmann et al. [2017] have shown that AGN-induced star formation quenching is most pronounced in massive galaxies. There is also compelling evidence that the accompanying RGs provide the energy necessary to stop (e.g. McNamara & Nulsen [2012], Yang et al. [2019]) bremsstrahlung-mediated cooling flows (Fabian et al. [2019]).
in clusters of galaxies. In the absence of cooling flows, the intra-cluster medium (ICM) remains dilute and hot, and galaxies in the centres of clusters are denied of infalling gas that could otherwise reignite star formation. Cosmological simulations that include this RG feedback indeed resolve (e.g. Croton et al. 2006) the overprediction of baryonic masses and luminosities of central cluster galaxies that early simulations found. Finally, RGs may be responsible for magnetising the IGM that pervades the filaments of the Cosmic Web (e.g. Vazza et al. 2017).

Despite the emerging picture that RGs trace quenched star formation, inhibit cooling flows and magnetise filaments, our knowledge of them is far from complete. Concerning geometry, major open questions are: What is the connection between the morphology of RGs and the pressure field of the ambient IGM, especially in filaments and cluster outskirts? Do small and large RGs come from the same initial population, or is their growth driven by distinct physical processes? How large can RGs become, and, more generally, how many RGs are there of each length? To test RG growth models that answer these and other questions, it is imperative to study the subpopulation of most spatially extreme RGs: the giant radio galaxies (GRGs). The defining feature of GRGs is that their proper lengths — when projected — are extremely large RGs: the largest RGs. The GRG phenomenon and the detection biases inherent to systematic search campaigns.

2. Theory

To measure the intrinsic GRG length distribution, we must first establish a suitable statistical framework. In this section, we provide a summary of the theory developed in Appendices A. Following Occam’s razor, we construct a model with minimal assumptions that provides new insight into the GRG phenomenon.

2.1. Intrinsic proper length

Firstly, we assume that GRGs and non-giant RGs share a common length distribution. In particular, because power laws are abundant in Nature, we assume that the intrinsic proper length random variable (RV) \( L \) has a Pareto distribution with tail index \( \xi \) and support from \( l_{\text{min}} \) onwards. If an RV is Pareto distributed, then the relative occurrence of two possible outcomes equals their ratio raised to a power: the tail index \( \xi \). In astrophysics, Pareto distributions describe the kinetic energies of freshly accelerated electrons in large-scale structure and supernova shocks (e.g. Kirk & Schneider 1987), the initial masses of main-sequence stars (e.g. Kroupa 2001), and the luminosities of gamma-ray bursts (e.g. Bloom et al. 2001), to name a few examples. Previous works (e.g. Andernach et al. 2021) have already hinted at the approximate validity of a Pareto distribution for GRG lengths. By comparing our final model to observations, as discussed in Section 4 and visualised in Fig. 17, we will demonstrate that this assumption is indeed a powerful approximation in the current case.

The probability density function (PDF) \( f_L : \mathbb{R} \to \mathbb{R}_{\geq 0} \) thus becomes

\[
f_L(l) = \begin{cases} 
0 & l \leq l_{\text{min}}, \\
\frac{\xi \left( \frac{l}{l_{\text{min}}} \right)^{-\xi}}{l_{\text{min}}^\xi} & l > l_{\text{min}}.
\end{cases}
\]  

(1)

We refer the reader to Appendix A.1 for a derivation of this expression and a demonstration of its connection to the literature’s most common parametrisation.

2.2. Projected proper length

From the distribution of intrinsic lengths and the assumption of random radio galaxy orientations, we now derive the distribution of projected lengths. This distribution is more easily compared to observations, which usually lack inclination angle information.

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1 An object’s size can refer to either its one-, two- or three-dimensional extent. We therefore consider ‘angular size’ to be ambiguous terminology; we propose that ‘angular length’ better captures one-dimensional extent. We therefore consider ‘angular size’ to be ambiguous terminology; we propose that ‘angular length’ better captures one-dimensional extent. Naturally, an object’s ‘length’ is understood to be its total one-dimensional extent, so that the qualifier ‘largest’ seems superfluous. We further remark that ‘length’ is synonymous with, but more succinct than, ‘linear size’. In cosmology, one must distinguish between proper and comoving lengths, especially when objects are not gravitationally bound — like GRG lobes. In this work, we consider two types of proper lengths: intrinsic proper lengths and projected proper lengths.

2 Luckily, this assumption will turn out to be irrelevant in the forthcoming GRG-only expressions, which are this work’s focus.
2.2.1. Distribution for RGs

To model length and orientation, we consider a vector $L \in \mathbb{R}^3$ (of length $L = ||L||_2$) for each RG. In accordance with the IAU Solar System convention for positive poles, the unit vector $\hat{L} \in \mathbb{S}^2$ marks the direction from which the central Kerr black hole is seen rotating in counterclockwise direction. We define the inclination angle $\Theta$ as the angle between $\hat{L}$ and a vector parallel to the line of sight pointing towards the observer. We model RGs as line segments — as if they were 'thin sticks', with vanishing volumes — so that the projected length $L_p$ relates to $L$ and $\Theta$ through

$$L_p = L \sin \Theta.$$  \hfill (2)

We assume that $\hat{L}$ is drawn from a uniform distribution on $\mathbb{S}^2$, so that the PDF $f_0 : [0, \pi] \to \mathbb{R}_{\geq 0}$ becomes $f_0(\theta) = \frac{1}{2} \sin \theta$. Since $L$ and $\Theta$ are independent, we find in Appendix A.2.1 that the PDF of $L_p, f_{L_p} : \mathbb{R} \to \mathbb{R}_{\geq 0}$, is

$$f_{L_p}(l_p) = \begin{cases} \frac{1}{l_p^{\xi+1}} & \text{if } l_p \leq 0, \\ \frac{\xi}{l_p^\xi} \frac{1}{l_p^{\xi+1}} - \frac{1}{l_p^{\xi+1}} & \text{if } 0 < l_p \leq \min_l, \\ -\frac{1}{l_p^{\xi+1}} + \frac{1}{l_p^{\xi+1}} & \text{if } l_p > \min_l, \end{cases}$$  \hfill (3)

where

$$I(a, b) := \int_1^a \frac{n^\xi \, dn}{\sqrt{n^2 - b^2}} \text{ for } a > 0, |b| < 1.$$  \hfill (4)

Note that for $l_p > \min_l$, the projected proper length has a Pareto distribution with the same tail index as the intrinsic proper length distribution. We compare $f_1$ and $f_{L_p}$ in Fig. 1.

2.2.2. Distribution for GRGs

For GRGs specifically (i.e. RGs such that $L_p > l_p,GRG$, where $l_p,GRG$ is some constant threshold; in this work, $l_p,GRG := 0.7$ Mpc), the projected proper length distribution becomes a Pareto distribution with tail index $\xi$ again:

$$f_{l_p > l_p,GRG}(l_p) = \begin{cases} \frac{\xi}{l_p^{\xi+1}} & \text{if } l_p \leq l_p,GRG, \\ \frac{1}{l_p,GRG} & \text{if } l_p > l_p,GRG. \end{cases}$$  \hfill (5)

Said differently, for GRGs, projection retains the Pareto-ness of lengths. A measurement of the tail index of the projected length distribution is immediately also a measurement of the tail index of the intrinsic length distribution.

The survival function, which gives the probability that a GRG has a projected proper length exceeding $l_p$, takes on a particularly simple form:

$$P(L_p > l_p | l_p > l_p,GRG) = \frac{l_p}{l_p,GRG}.$$  \hfill (6)

The mean projected proper length of GRGs is the expectation value of $L_p | L_p > l_p,GRG$:

$$E[L_p | L_p > l_p,GRG] = \frac{\xi+1}{\xi+2} l_p,GRG,$$  \hfill (7)

which is only defined when $\xi < -2$. For example, when $\xi = -4$, $E[L_p | L_p > l_p,GRG] = \frac{1}{2} l_p,GRG$, which becomes 1.05 Mpc for $l_p,GRG \approx 0.7$ Mpc and 1.5 Mpc for $l_p,GRG \approx 1$ Mpc.

Appendix A.2.2 provides a derivation for all three expressions.

2.3. Deprojection factor

The deprojection factor, $D := \frac{L}{l_p} = \sin^{-1} \Theta$, quantifies how much larger intrinsic lengths are compared to projected lengths. The PDF of $D, f_D : \mathbb{R} \to \mathbb{R}_{\geq 0}$, is

$$f_D(d) = \begin{cases} 0 & \text{if } d \leq 1; \\ \frac{1}{d^2} & \text{if } d > 1. \end{cases}$$  \hfill (8)

The mean deprojection factor $E[D] = \frac{1}{\xi}$. Deprojection factors can become arbitrarily large under the current model, because projected lengths can become arbitrarily small. As discussed in Sect. 5.2, this is not a very realistic set-up. In reality, an RG’s projected length is bounded from below by its lobes, which have a non-vanishing volume and thus extend along all three spatial dimensions. Upon projection, the projected length therefore cannot shrink beyond some lower limit that depends on the lobe geometry. In Appendix A.3, we show that by enriching the conventional stick-like geometry with spherical lobes, deprojection factors indeed become bounded.
2.4. Intrinsic proper length posterior and its moments

Because an RG’s intrinsic length is more physically informative than its projected length, we ideally obtain the former. In this subsection, we quantify what a measurement $L_p = l_p$ already reveals about $L$.

We first note that the projected length bounds the intrinsic length from below. The intrinsic length can be much larger, however, but this is improbable for two reasons: large lengths are rarer than small lengths, although how drastic this effect is depends on $\xi$; in addition, viewing directions with large inclination angles are uncommon. The best we can do is to construct a posterior distribution for $L$ given $L_p = l_p$. This posterior has a concise analytic form. If $l_p > l_{\text{min}}$, which is the relevant case for GRGs, the distribution of $L | L_p = l_p$ is

$$ f_{L|L_p=l_p}(l) = \begin{cases} 0 & \text{if } l \leq l_p \\ \frac{-\xi}{2^{\xi+1} \pi} \left( \frac{2^\xi}{\Gamma(\xi+1)} \right) \frac{1}{l^{\xi+1}} (l_p/l)^\xi & \text{if } l > l_p. \end{cases} \quad (9) $$

For $l \gg l_p$, $f_{L|L_p=l_p}(l) \propto (l/l_p)^{-2}$: the posterior PDF tends to a power law in $l$ with index $\xi - 2$. In Fig. 2 we visualise the posterior PDF for several values of $\xi$. Clearly, to evaluate Eq. (9), one must choose $l_p$ — however, the shape of the distribution is the same for all choices. We illustrate this by comparing the PDF for a comparatively small GRG ($l_p = 1.0$ Mpc; top panel) to the PDF for Alcyoneus ($l_p = 5.0$ Mpc; bottom panel).

The posterior mean is

$$ \mathbb{E}[L | L_p = l_p] = l_p \cdot \frac{-\xi}{2^{\xi+1} \pi} \left( \frac{2^\xi}{\Gamma(\xi+1)} \right) \frac{1}{l_p^{\xi+1}} (l_p/l)^\xi. \quad (10) $$

the posterior variance is

$$ \mathbb{V}[L | L_p = l_p] = l_p^2 \left( \frac{-\xi}{\xi + 1} - \frac{\xi^2}{2^{\xi+2} \pi^2} \Gamma^2(\xi) + \frac{\xi}{\Gamma(\xi+1)} \right). \quad (11) $$

Both mean and standard deviation scale linearly in $l_p$; the projection effect is a multiplicative noise source. In Table 1 we provide explicit values for various $\xi$.

**Table 1:** Intrinsic proper length posterior mean and standard deviation in multiples of projected proper length $l_p$, given for various tail indices $\xi$.

| $\xi$  | $\mathbb{E}[L | L_p = l_p]$ (multiples of $l_p$) | $\mathbb{V}[L | L_p = l_p]$ (multiples of $l_p^2$) |
|--------|-----------------------------------------------|-----------------------------------------------|
| $-2$   | $\frac{3}{4}$ ≈ 1.27                         | $\frac{2}{\pi}$ ≈ 0.62                        |
| $-3$   | $\frac{4}{5}$ ≈ 1.18                         | $\frac{1}{5}$ – $\frac{5\pi^2}{24}$ ≈ 0.33    |
| $-4$   | $\frac{32}{63}$ ≈ 1.13                        | $\frac{4}{5}$ – $\frac{1024}{81\pi^2}$ ≈ 0.23 |
| $-5$   | $\frac{45\pi}{128}$ ≈ 1.10                   | $\frac{5}{4 \pi}$ – $\frac{2025\pi^2}{16388}$ ≈ 0.17 |

Higher moments exist up to order $[-\xi]$; because the PDF $f_{L|L_p=l_p}(l)$ is strongly skewed, such moments do further specify the distribution. It is important to note that it is formally incorrect to statistically deproject RGs by drawing samples from deprojection factor $D$ and multiplying them with some measurement $L_p = l_p$, or even more crudely, by multiplying the latter with $\mathbb{E}[D]$. The reason that renders such approaches invalid is that $L_p = L \sin \Theta$ and $D = \sin^{-1} \Theta$ are not independent RVs. We refer to Appendix A.4 for an explicit proof of this fact, and for derivations of this subsection’s expressions.

![Graph showing the probability density for Alcyoneus](image)

**Fig. 2:** If tail index $\xi$ is known, an RG’s projected proper length $l_p$ fixes the probability distribution over its possible intrinsic proper lengths. This distribution is strongly skewed, and the same for all $l_p$ — save for horizontal translation and vertical scaling. We illustrate this point by showing posterior PDFs for GRGs with two different projected proper lengths. Top: $l_p = 1$ Mpc. Bottom: $l_p = 5$ Mpc. For $\xi = -4$, the posterior mean $\mathbb{E}[L | L_p = l_p] = 1.13 l_p$ and the posterior standard deviation $\sqrt{\mathbb{V}[L | L_p = l_p]} = 0.23 l_p$ (see Table 1).

2.5. GRG inclination angle

Radio galaxies with jets that make a small angle with the plane of the sky are more likely to have a projected length exceeding $l_{p,\text{GRG}}$ than those with jets that make a large angle with the plane of the sky. For this reason, the inclination angle distribution of GRGs is different from that of RGs: it is more peaked around $\theta = 90^\circ$. More precisely, the PDF $f_\Theta | L_p > l_{p,\text{GRG}} : [0, \pi] \to \mathbb{R}_{\geq 0}$ of the GRG inclination angle $\Theta | L_p > l_{p,\text{GRG}}$ has the general
Fig. 3: GRGs have a different inclination angle distribution than RGs. GRGs more often have orientations close to the sky plane. The strength of this tendency is governed by $\xi$, with larger $\xi$ meaning more dispersion. For comparison, in grey, we show the RG inclination angle distribution.

$$f_\theta | L_p > l_{p,GRG} (\theta) = \frac{1 - F_L \left( \frac{l_{GRG}}{\sin \theta} \right)}{1 - F_{L_p} (l_{p,GRG})}. \quad (12)$$

Under our Pareto distribution assumption for $L$, this concretises to

$$f_\theta | L_p > l_{p,GRG} (\theta) = \frac{2 \Gamma \left( -\frac{\xi}{2} + 1 \right)}{(-\xi + 1) \sqrt{\pi} \Gamma \left( -\frac{\xi + 1}{2} \right)} \sin^{-\xi} \theta. \quad (13)$$

We note that $f_\theta | L_p > l_{p,GRG} (\theta) \propto \sin^{-\xi} \theta$; the factor in front serves only as a normalisation constant. The distribution is independent of the choice of $l_{p,GRG}$ and depends on a single parameter: $\xi$. We visualise $f_\theta | L_p > l_{p,GRG} (\theta)$ in Fig. 3. Appendix A.3 contains a brief derivation.

2.6. GRG angular length

The model predicts the distribution of GRG angular lengths in the Local Universe up to comoving distance $r_{\text{max}}$. The GRG angular length RV $\Phi$ if $L_p > l_{p,GRG}$ relates to the GRG projected proper length RV $L_p | L_p > l_{p,GRG}$ and the comoving distance RV $R$ as

$$\Phi | L_p > l_{p,GRG} = L_p | L_p > l_{p,GRG} \frac{1 + z (R)}{R}. \quad (14)$$

(Note that this relation is valid only in a flat Friedmann–Lemaître–Robertson–Walker (FLRW) universe.) We also assume that the GRG number density is constant in the Local Universe. The PDF of $\Phi | L_p > l_{p,GRG}$ has useful analytic forms under two different idealisations.

In a Euclidean universe, $z(R) = 0$, and the minimal GRG angular length $\phi_{\text{GRG}} = \frac{l_{GRG}}{r_{\text{max}}}$. Then

$$f_{\phi} | L_p > l_{p,GRG} (\phi) = \begin{cases} 0 & \text{if } \phi \leq \phi_{\text{GRG}} \\ \frac{3 \xi + 1}{\xi + 4} \left( \frac{\phi}{\phi_{\text{GRG}}} \right)^{\xi} & \text{if } \phi > \phi_{\text{GRG}}, \end{cases} \quad (15)$$

which is valid as long as $\xi \neq -4$.

In an expanding universe at low redshifts, the Hubble–Lemaître law $z(R) \approx \frac{\xi}{d_H}$ holds; the Hubble distance $d_H \approx \frac{c}{H_0}$. In this case,

$$f_{\phi} | L_p > l_{p,GRG} (\phi) = \begin{cases} 0 & \text{if } \phi \leq \phi_{\text{GRG}} + \frac{l_{p,GRG}}{d_H} \\ \frac{-3(\xi + 1)d_{\text{GRG}}}{r_{\text{max}} + l_{p,GRG}} \frac{\phi}{d_H} & \text{if } \phi > \phi_{\text{GRG}} + \frac{l_{p,GRG}}{d_H}. \end{cases} \quad (16)$$

Figure 4 shows GRG angular length PDFs under both idealisations. The PDFs undergo a minor shift upon changing universe type but are otherwise similar. For most current-day applications, it will therefore be unnecessary to calculate an even more refined version of $f_{\phi} | L_p > l_{p,GRG} (\phi)$. Appendix A.4 contains derivations and details.

2.7. Maximum likelihood estimation of the tail index

The GRG projected proper length distribution features just one parameter of physical interest: the tail index $\xi$. If observational selection effects are negligible, one can directly use maximum likelihood estimation (MLE) on GRG data to infer $\xi$. In particular, we consider a set of projected lengths $|\hat{L}_p, \ldots, \hat{L}_p| < \hat{L}_p | L_p > l_{p,GRG}$ from $N$ GRGs. Appendix A.5 shows that the maximum likelihood estimate of $\xi$ is the RV $\hat{\xi}_{\text{MLE}}$, given by

$$\hat{\xi}_{\text{MLE}} = -\frac{N}{\sum_{i=1}^{N} \ln \frac{\hat{L}_p}{l_{p,GRG}}} - 1. \quad (17)$$

2.8. Observed projected proper length

2.8.1. General considerations

In the preceding theory, we have ignored observational selection effects that favour some projected proper lengths over others. In practice, several such effects occur; the importance of

Article number, page 5 of 68
each varies per survey and (G)RG search campaign within it. One of them is the bias against physically long RGs that the interferometer’s largest detectable angular scale can induce\footnote{For example, the Faint Images of the Radio Sky at Twenty Centimeters (FIRST) survey used the Very Large Array (VLA) in B-configuration, leading it to detect angular scales of at most two arcminutes. By contrast, the largest angular scale of the LoTSS — the survey relevant to this work — is about a degree. (For the 60’ and 90’ resolutions, the shortest baseline is 100 metres; for the 60’ and 90’ resolutions, the shortest baseline is 68 metres.) As virtually all GRGs are of subdegree angular length, we need not consider this bias in our case.}. As a result, the projected proper length of an observed RG might not be adequately modelled through RV \( L_p \). Instead, we must introduce a new RV \( L_{p, \text{obs}} \).

We define the completeness \( C : \mathbb{R}_{+0} \times \mathbb{R}_{+0} \rightarrow [0, 1] \) at \((l_p, z_{\text{max}})\) to be the fraction of all RGs with projected proper length \( l_p \) in the cosmological volume up to \( z_{\text{max}} \) that is detected in a particular RG search campaign. Then, assuming that the distribution of \( L_p \) does not evolve with redshift between \( z = z_{\text{max}} \) and \( z = 0 \) (i.e. \( \xi \) remains constant),

\[
C(l_p, z_{\text{max}}) = \frac{\int_0^{z_{\text{max}}} p_{\text{obs}}(l_p, z) r^2(z) E^{-1}(z) \, dz}{\int_0^{z_{\text{max}}} r^2(z) E^{-1}(z) \, dz},
\]

(18)

where \( p_{\text{obs}}(l_p, z) \) is the probability that an RG of projected proper length \( l_p \) at cosmological redshift \( z \) is detected through the campaign, and \( r(z) \) is the comoving radial distance at cosmological redshift \( z \). In a flat FLRW universe, the dimensionless Hubble parameter \( E \) is

\[
E(z) := \frac{H(z)}{H_0} = \sqrt{\Omega_{K, 0}(1 + z)^3 + \Omega_{M, 0}(1 + z)^3 + \Omega_{\Lambda, 0}}.
\]

(19)

The PDF of the observed projected proper length RV \( L_{p, \text{obs}} \) becomes

\[
f_{L_{p, \text{obs}}}(l_p) = \frac{C(l_p) f_{L_p}(l_p)}{\int_0^{z_{\text{max}}} C(l_p) f_{L_p}(l_p) \, dl_p},
\]

(20)

where we suppress the \( z_{\text{max}} \)-dependence for succinctness. Note that multiplying \( p_{\text{obs}}(l_p, z) \) with an \( l_p \) and \( z \)-independent factor affects the completeness \( C(l_p, z_{\text{max}}) \), but cancels in Eq. \( 20 \); \( f_{L_{p, \text{obs}}} \) will be independent of it. Finally, the PDF of the GRG observed projected proper length RV \( L_{p, \text{obs}} \) at \( L_{p, \text{obs}} > l_p, \text{GRG} \) is

\[
f_{L_{p, \text{obs}} \mid L_{p, \text{obs}} > l_p, \text{GRG}}(l_p) = \begin{cases} 0 & \text{if } l_p \leq l_p, \text{GRG} \\ \frac{C(l_p) f_{L_p}(l_p)}{L_p, \text{GRG}}(l_p) \, dl_p & \text{if } l_p > l_p, \text{GRG}. \end{cases}
\]

(21)

We derive these expressions in Appendix A.8.1.  

2.8.2. Fuzzy angular length threshold

We provide a concrete example of an important observational selection effect in visual searches for GRG candidates in survey images. To cope with the sheer number of detectable RGs in modern surveys like the LoTSS DR2, a natural criterion is to only add sources to a candidate list if they appear — by eye — to have an angular length larger than some threshold. However, it is hard to precisely assess the angular length of a candidate before actually measuring it; sometimes, a candidate with a smaller angular length than the threshold will feature in the list, while some candidates with a larger angular length than the threshold will not. This leads to the notion of a ‘fuzzy angular length threshold’, where the probability that an RG with angular length \( \phi \) is observed through the visual search increases (e.g. linearly) from 0 to 1 between \( \phi_{\text{min}} \) and \( \phi_{\text{max}} \):

\[
p_{\text{obs, AL}} (l_p, z) = \min \left\{ \max \left( \phi (l_p, z) - \phi_{\text{min}}, 0 \right), 1 \right\},
\]

(22)

\[
\phi (l_p, z) = \frac{L_p (1 + z)}{r(z)}.
\]

(23)

See the top row of Fig. 5 for several examples of associated completeness curves \( C(l_p) \) and observed projected proper length PDFs. See Appendix A.8.2 for additional information.

2.8.3. Surface brightness limitations

Another important observational selection effect is due to a survey’s finite noise level. The noise determines the surface brightness threshold \( b_{\text{th}} \) (typically comparable to the noise level itself) below which radio galaxy features remain visually undetected.

Fanaroff–Riley class II We model the lobe surface brightness RV \( B_s \) at the central observing frequency \( \nu_{\text{obs}} \) as

\[
B_s = \frac{b_{\nu, \text{ref}} \cdot S}{(1 + Z)^{-\alpha}},
\]

(24)

where \( b_{\nu, \text{ref}} \) is the median surface brightness of RGs of intrinsic proper length \( l_{\text{ref}} \) at cosmological redshift \( z = 0 \) and frequency \( \nu_{\text{obs}} \), and \( S \) is a lognormally distributed RV with median 1 and dispersion parameter \( \sigma_{\nu, \text{ref}} \) that captures the variability in surface brightness among this population of RGs. The denominator models the fact that surface brightness is not conserved with distance in a FLRW universe; \( Z \) is the cosmological redshift RV up to \( z = z_{\text{max}} \) and \( \alpha \) is the typical lobe spectral index. The exponent \( \xi < 0 \) characterises the scaling between intrinsic proper length and surface brightness. (If RGs are self-similar, so that morphology does not predict length, one finds \( \xi = -2 \).) For this selection effect, the observing probability is

\[
p_{\text{obs, SB}} (l_p, z) = \int_{s_{\text{min}}}^{s_{\text{max}}} \sqrt{1 - \left( \frac{s_{\text{min}}}{s} \right)^2} f_s (s) \, ds,
\]

(25)

\[
s_{\text{min}} = \frac{b_{\nu, \text{th}}}{b_{\nu, \text{ref}}} \left( \frac{l_p}{l_{\text{ref}}} \right)^{\frac{\alpha}{3}} (1 + Z)^{3-\alpha},
\]

(26)

\[
f_s (s) = \frac{1}{2 \sqrt{\pi} \sigma_{\nu, \text{ref}}} \exp \left( -\frac{\ln^2 s}{2 \sigma_{\nu, \text{ref}}^2} \right).
\]

(27)

Note that \( p_{\text{obs, SB}} \) does not depend on \( b_{\text{th}} \) or \( b_{\nu, \text{ref}} \) separately, but on their ratio only. See the bottom row of Fig. 5 for several examples of associated completeness curves \( C(l_p) \) and observed projected proper length PDFs.

Fanaroff–Riley class I For FRI RGs, the assumption of a constant surface brightness beyond the core is inaccurate. The simplest correction in which FRI RGs retain a well-defined notion of length assumes a linearly decreasing surface brightness profile, which peaks at the core and goes to zero at the RG’s two endpoints. (A power-law profile does not work: in such case, the surface brightness only goes asymptotically to zero — but never
Fig. 5: Selection effects leave imprints on the distribution of radio galaxies’ observed projected proper lengths. In the top row, we show how imposing an angular length threshold in a GRG search campaign leads to incompleteness (left), which causes the PDF \( f_{L_{\text{obs}}} \) (right) to differ from the PDF \( f_{L} \). We assume RGs with angular length \( \phi < \phi_{\text{min}} \) have probability 0 to be included in a sample, whilst RGs with angular length \( \phi > \phi_{\text{max}} \) have probability 1. The inclusion probability is assumed to increase linearly between \( \phi_{\text{min}} \) and \( \phi_{\text{max}} \). In the left panel, we fix \( \phi_{\text{min}} = 3' \) and vary \( \phi_{\text{max}} \); in the right panel, we also fix \( \phi_{\max} = 7' \). In the bottom row, we show how surface brightness limitations of a survey lead to incompleteness (left), which causes the PDF \( f_{b_{\text{obs}}} \) (right) to differ from the PDF \( f_{b} \). We assume that lobe surface brightnesses are lognormally distributed; we parametrise the distribution for RGs of intrinsic length \( L_{\text{obs}} = 0.7 \text{ Mpc} \) at \( z = 0 \) observed at \( \nu_{\text{obs}} = 144 \text{ MHz} \) with a median \( b_{\text{obs},\nu_{\text{ref}}} \) and dispersion parameter \( \sigma_{\nu_{\text{ref}}} \).

In the left panel, we fix \( \sigma_{\text{ref}} = 1.5 \) and vary \( b_{\text{ref},\nu} \); in the right panel, we also fix \( b_{\text{ref},\nu} = 1000 \text{ Jy deg}^{-2} \). We assume a lobe spectral index \( \alpha = -1 \), a surface brightness detection threshold \( b_{\text{th},\nu} = 25 \text{ Jy deg}^{-2} \), and self-similar growth: \( \zeta = -2 \). For both selection effects, we consider RGs up to cosmological redshift \( z_{\text{max}} = 0.5 \) only.

Actually reaches it.) In this case, we define RV \( B_{\ell} \) to be the mean surface brightness along an RG's jets, which can be regarded as a typical value for that RG. As \( B_{\ell} \) again obeys Eq. 24, we find that the formulaic structure of \( p_{\text{obs},\nu}(l_{\nu}, z) \) is identical for FRI and FRII GRGs, except that FRII GRGs require a change

\[
b_{\text{th},\nu} \rightarrow \frac{b_{\text{th},\nu}}{2 \left( 1 - \frac{b_{\text{GRG},\nu}}{b_{\text{th},\nu}} \right)},
\]

which affects \( p_{\text{obs},\nu}(l_{\nu}, z) \) through \( \sigma_{\text{min}} \). There is no change for \( l_{\nu} = 2l_{\text{GRG},\nu} \).

Although the formulaic structure might be the same, the best-fit parameters can differ. For example, it is possible that \( \sigma_{\text{ref,FRI}} \neq \sigma_{\text{ref,FRII}} \) or \( \zeta_{\text{FRI}} \neq \zeta_{\text{FRII}} \). See Appendix A.8.3 for derivations and numerical implementation considerations. To include both aforementioned selection effects at the same time, a natural approximation is to assume that the observing probability simply factorises:

\[
p_{\text{obs}}(l_{\nu}, z) \approx p_{\text{obs,AL}}(l_{\nu}, z) \cdot p_{\text{obs,SB}}(l_{\nu}, z).
\]

2.9. GRG number density

A central question in the field of radio galaxies is: how intrinsically rare are GRGs? By counting GRGs in a search campaign with well-understood selection effects, we can give an answer. More precisely, one can estimate the comoving number density of GRGs in the Local Universe, \( n_{\text{GRG}} \), through the number of observed GRGs up to cosmological redshift \( z_{\text{max}} \) in a uniformly searched region of sky of solid angle \( \Omega \), \( N_{\text{GRG,obs}}(z_{\text{max}}, \Omega) \). Then, under the standard assumption \( l_{\nu,\text{GRG}} > l_{\text{min}} \), Appendix A.9 shows that

\[
n_{\text{GRG}} = \frac{l_{\nu,\text{GRG}}^{\xi+1}}{\pi H_{\nu} c} \cdot \int_{z_{\text{min}}}^{z_{\text{max}}} \int_{l_{\text{GRG}}}^{\infty} N_{\text{GRG,obs}}(z_{\text{max}}, \Omega) \theta_{\text{GRG}}(l_{\nu}, z) 4\pi r^{2}(z) E^{-1}(z) dz dl_{\nu}.
\]

Although the appropriate \( p_{\text{obs}}(l_{\nu}, z) \) varies per search campaign, it is always possible to bound \( p_{\text{obs}}(l_{\nu}, z) \) from above — e.g. by 1. In such case, Eq. 30 bounds \( n_{\text{GRG}} \) from below.

2.10. GRG lobe volume-filling fraction

Because giants attain cosmological lengths, they might contribute to the energisation and magnetisation of Cosmic Web filaments in regions that smaller radio galaxies cannot reach. A key statistic that measures the enrichment of the Cosmic Web by giants is the volume-filling fraction (VFF) of their lobes. Assuming that lobes do not grow along with the expansion of the Universe, the proper VFF changes over cosmic time:
An alternative is to deviate slightly from our Pareto ansatz and plain the observational diversity of active galaxies. It posits that AGN with quasar appearance (quasar RGs) have inclination angles \( \theta \leq \theta_{\text{max}} \) or \( \theta \geq 180^\circ - \theta_{\text{max}} \) and giants generated by AGN without quasar appearance (non-quasar GRGs) have intermediate \( \theta \), we can predict the probability \( p_Q \) that an observed giant is a quasar GRG. As long as quasar GRGs and non-quasar GRGs are subject to the same selection effects, these selection effects do not affect \( p_Q \). Instead, in such case, \( p_Q \) only depends on the maximum quasar inclination angle \( \theta_{\text{max}} \) and the tail index \( \xi \) of the Paretoian intrinsic proper length distribution.

If quasar RGs are more closely aligned with the line of sight than non-quasar RGs but are otherwise similar, fewer of them will satisfy \( \theta \geq \theta_{\text{max}} \) and thus be classified as GRGs. Therefore, the quasar GRG fraction \( f_Q \) — the fraction of quasar GRGs in an actual GRG sample — constrains \( \theta_{\text{max}} \). We model \( f_Q \) as an RV:

\[
f_Q \approx \frac{N_Q}{N}, \quad N_Q \sim \text{Binom}(N, p_Q),
\]

where the RV \( N_Q \) is the number of quasar GRGs in the sample, the constant \( N \) is the total number of GRGs in the sample and the parameter \( p_Q \) is the quasar GRG probability. Our framework predicts

\[
p_Q = \frac{\mathbb{P}(L_{\text{obs}} \geq l_{p,\text{GRG}} \mid \sin \Theta \leq \sin \theta_{\text{max}})}{\mathbb{P}(L_{\text{obs}} \geq l_{p,\text{GRG}})} = \frac{4 \int_{\theta_{\text{max}}}^{\pi} \sin \Theta \left( \Gamma \left(-\frac{\xi}{2} + 1\right) \right) \sin^{\frac{\xi}{2}} \left( \Gamma \left(-\frac{\xi}{2} - \frac{1}{2}\right)\right) \sqrt{1 - x^2} \, dx}{\left(\xi + 1\right)^{\frac{\xi}{2}} \sqrt{\Gamma \left(-\frac{\xi}{2} - \frac{1}{2}\right)}}.
\]

See Appendix A.11 for a derivation. Interestingly, as long as quasar GRGs and non-quasar GRGs are subject to the same selection effects, these selection effects do not affect \( p_Q \). Figure 6 shows that, for all relevant \( \xi, p_Q \) is a steeply and monotonically increasing function of \( \theta_{\text{max}} \). Thus, knowing \( \xi \), one can use an empirical \( f_Q \) to determine \( p_Q \) and in turn \( \theta_{\text{max}} \).

2.12 Extreme GRGs in a sample

The model can predict the occurrence of GRGs with extreme projected proper lengths in a sample of cardinality \( N \). The prob-
3. Sample compilation and properties

To measure the intrinsic GRG length distribution, we also require a large sample of GRGs collected from a single survey through a systematic approach. This ensures approximately homogeneous selection effects, which we can correct for in subsequent analysis using the statistical framework of Sect. 2.

3.1. LoTSS DR2

The Low-Frequency Array (LOFAR; Van Haarlem et al. 2013) is a powerful, Pan-European radio interferometer that features both (sub)arcsecond-scale resolution and sensitivity to degree-scale structures. Dabhade et al. (2020b) have already demonstrated that this combination is ideal for detecting GRGs: these authors found a record 225 new specimens in the LoTSS DR1, the first data release of the LOFAR’s Northern Sky survey at central frequency $\nu_{\text{obs}} = 144 \text{ MHz}$. Excitingly, the recent LoTSS DR2 (Shimwell et al. 2022) improves the data calibration and increases the survey footprint from 424 deg$^2$ to 5635 deg$^2$ — i.e. by more than a factor 13. By default, the LoTSS features imagery at 60’ and 20’ resolutions. To further facilitate the discovery of GRGs (among other goals), we reprocess the LoTSS DR2 by subtracting compact sources and imaging at 60’ and 90’ resolution; more details are given in Oei et al. (2022a) and Oei et al. (in prep.a). This 60’ and 90’ imagery has turned out to be effective in highlighting jets and lobes of RGs of large angular and physical extent, whose surface brightnesses are low and which therefore have remained undetected in shallower surveys, and even in the LoTSS DR2 at higher resolutions. We demonstrate this fact in Fig. 8 by comparing the LoTSS DR2 at 60’ and 90’ for three GRGs whose discovery has relied on the lower-resolution images. After the serendipitous discovery of several such hitherto unknown GRGs in the 60’ and 90’ images, we decided to initiate a systematic, multi-resolution, visual GRG search through the entire LoTSS DR2 area. This search comprised of a hundreds-of-hours-long inspection of the LoTSS DR2 maps at 60’ and 90’, alongside Pan-STARRS DR1 (Chambers et al. 2016) and SDSS DR9 (Ahn et al. 2012) maps, in Aladin Desktop 11.0 (Bonnarel et al. 2000). Reliable automated search strategies do not yet exist for several reasons. Giants showcase a rich morphological variety (see Sect. 3.3) and are easily confused with other types of astrophysical sources (see Sects. 3.3 and 3.4). Moreover, the known population is too small to effectively apply supervised learning techniques. However, it appears possible to find giants in morphological outlier lists of unsupervised learning techniques such as self-organising maps (SOMs; Mostert et al. 2021). The efficacy of such techniques in GRG searches has not yet been quantified. 

3.2. Angular length threshold

To limit the amount of manual work, we decided to search only for GRG candidates whose angular length exceeds some threshold. In Fig. 9 we show how the angular length $\phi$ of GRGs with six different projected proper lengths $l_p$ varies as a function of cosmological redshift $z$. Because of the expansion of the Universe, GRGs have an arcminute-scale minimum angular length: if $l_p,GRG = 0.7 \text{ Mpc}$, all GRGs obey $\phi > 1.3^\circ$. For the purpose
Fig. 8: By subtracting compact sources from calibrated 144 MHz visibility data and imaging at low resolution (60″ and 90″), we reveal otherwise speculative giant radio galaxies at the unexplored ~1 Jy deg⁻² surface brightness level. We compare LoTSS DR2 cutouts of three newly discovered GRGs at 6″ (left panels) and 60″ (right panels). The claimed host galaxy is in the image centre. Top: a GRG of projected proper length \( l_p = 1.4 \pm 0.3 \) Mpc, whose angular length \( \phi = 32.3 \pm 0.2^\prime \) is larger than that of the full Moon. Middle: a GRG of \( l_p = 1.6 \pm 0.6 \) Mpc and \( \phi = 16.4 \pm 0.2^\prime \). Bottom: a GRG of \( l_p = 3.6 \pm 0.1 \) Mpc and \( \phi = 8.5 \pm 0.2^\prime \). For scale, we show the stellar Milky Way disk (with a diameter of 50 kpc) generated using the Ringermacher & Mead (2009) formula, alongside a 3, 5, or 10 times inflated version.
GRGs of particular projected proper length

- $l_p = 0.7\,\text{Mpc} \ | \ \phi > 1.3'$
- $l_p = 1.0\,\text{Mpc} \ | \ \phi > 1.9'$
- $l_p = 2.0\,\text{Mpc} \ | \ \phi > 3.8'$
- $l_p = 3.0\,\text{Mpc} \ | \ \phi > 5.8'$
- $l_p = 4.0\,\text{Mpc} \ | \ \phi > 7.7'$
- $l_p = 5.0\,\text{Mpc} \ | \ \phi > 9.6'$

Fig. 9: As a function of redshift, how large on the sky do GRGs of particular projected proper lengths appear?

We show the relation between angular length $\phi$ and cosmological redshift $z$ for GRGs of six different projected proper lengths $l_p$. Due to the expansion of the Universe, there is a minimum angular length for each $l_p$. If one defines GRGs as RGs with $l_p \geq l_p^\text{GRG} = 0.7\,\text{Mpc}$, all GRGs have an angular length of $1.3'$ or above. If one instead defines GRGs as RGs with $l_p \geq l_p^\text{GRG} = 1\,\text{Mpc}$, all GRGs have an angular length of $1.9'$ or above.

of finding a GRG, it is therefore never useful to inspect a source with an angular length less than $1.3'$. Our highest priority has been to find GRGs with $z < 0.2$, which lie in a volume for which the total matter density field is known or will be known in the coming years through the combined power of deep spectroscopic surveys and Bayesian inference frameworks, such as the Bayesian Origin Reconstruction from Galaxies (BORG; Jasche & Wandelt 2013; Jasche et al. 2015; Jasche & Lavaux 2019). In an upcoming publication, we combine a sample of low-redshift GRGs with the BORG to analyse the large-scale environments of GRGs (Oei et al. in prep.). Figure 9 shows that all GRGs with $l_p > 1\,\text{Mpc}$ at $z < 0.2$ have an angular length $\phi > 5'$. For this reason, we have chosen $5'$ as the angular length threshold of our search campaign. This choice has kept the visual inspection duration to order $\sim 10^4\,\text{h}$, while still enabling us to target all Mpc-exceeding GRGs in the Local Universe ($z < 0.2$). In practice, this threshold is ‘fuzzy’: it is hard to accurately estimate angular lengths by eye before performing an actual measurement, so that our list of GRG candidates does contain some with a smaller angular length than the specified threshold. Inversely, it will likely lack some GRG candidates with an angular length exceeding the threshold.

3.3. GRG candidates in the radio

We first identify GRG candidates in the LoTSS DR2 at $6''$ and $60''$. These maps serve complementary roles. The $6''$ images reveal the precise morphology of radio galaxy cores and jets, which are necessary to pinpoint the host galaxy. Figure 10 provides a representative sense of the host galaxy identification accuracy these data allow for when combined with modern optical surveys such as the DESI Legacy Imaging Surveys (DESI Legacy Imaging Surveys DR9). In contrast, the $60''$ images have such compact sources removed or highly suppressed, but better highlight diffuse structures, such as RG lobes. Being similar in morphol-
Table 2: Excerpt of Table E.1, which details properties of the 2050 redshift spiral galaxies and their circumgalactic media, or radio halos and relics in galaxy clusters. Lying outside of the coverage, some GRG host galaxies have no SDSS DR12 name. Possible redshift types are $s$ (spectroscopic) and $p$ (photometric). We estimate the angular length errors to be 0.15′. Whether the host’s AGN has a quasar appearance is indicated by $y$ (yes) or $n$ (no). In cases where only a set of candidates containing the host galaxy could be established beyond reasonable doubt, we provide the redshift and projected proper length of the lowest-redshift candidate, which bounds the actual projected proper length from below. In such cases, to signify uncertainty, we mark the host name, host coordinates, and physical host properties in grey.

3.4. GRG candidates in the optical

To confirm that a radio structure really is a radio galaxy, we compare the radio images with optical images of the same sky region. If a patch of radio emission is indeed due to RG jets or lobes, the patch itself must have no cordirectional galactic counterpart in the optical. If the radio emission is due to a low-redshift spiral galaxy or a galaxy cluster instead, a corresponding easily recognisable counterpart will exist. We also take care not to erroneously associate the lobes of two distinct RGs. For this reason, we are more cautious to associate a pair of lobes to a suspected host galaxy when, in the optical, one can discern other galaxies in the angular vicinity of the lobes that could have generated them instead.

The Pan-STARRS and SDSS images used for these purposes complement each other, as they differ in quality throughout the sky — and in particular around sources of high optical flux density. Neither consistently outperforms the other. Only Pan-STARRS covers the full Northern Sky and could thus always be relied upon.

3.5. Host galaxy identification

We also used the Pan-STARRS and SDSS maps for the identification of host galaxies. We collected host redshifts from the SDSS DR12 (Alam et al. 2015) and Gaia (Gaia Collaboration et al. 2016) DR3 (Gaia Collaboration et al. 2021) through automated Vizier queries, from the Galaxy List for the Advanced Detector Era (GLADE) 2.4 (Daiya et al. 2019), and from the DESI DR9 photometric redshift catalogue (Zou et al. 2022). If redshifts from multiple sources were available, we favoured SDSS over GLADE data, GLADE over Gaia data, and Gaia over DESI data. Similarly, we only adopted photometric redshifts if spectroscopic ones were not available.

For a small subset of RGs, a definite host galaxy could not be established beyond reasonable doubt, but a set of candidates containing the host galaxy could. In these cases, the lowest-redshift candidate provides a lower bound to the RG’s projected proper length. If this lower bound exceeds $l_{\text{GRG}} = 0.7$ Mpc, then the actual projected proper length certainly does so too, and the RG can be classified as a GRG — despite persisting uncertainty concerning the identity of the host galaxy.

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12 A small fraction of observed RGs are double-double radio galaxies, which show four lobes.

13 This is true as long as all candidates have a redshift below that of the angular diameter distance maximum — $z = 1.59$ for the cosmology adopted.
3.6. LoTSS DR2 GRG sample

Our search campaign has led to the identification of 2050 hitherto unknown giants. To establish novelty, we assemble a literature catalogue with all known GRGs as of September 2022, combining the catalogue of Dabhade et al. (2020a) with the GRGs discovered in Galvin et al. (2020), Ishwara-Chandra et al. (2020), Tang et al. (2020), Bassani et al. (2021), Brüggen et al. (2021), Delhaize et al. (2021), Masini et al. (2021), Kuzmíc & Jamrozy (2021), Andernach et al. (2021), Mahato et al. (2022), Delhaize et al. (2021), and Gürkan et al. (2022), and Simonte et al. (2022). Fusing our sample with this literature catalogue, we obtain a final catalogue with N = 3342 GRGs.

Figure 11 shows the locations of all known giants in the sky. Table E.1 lists basic properties of all new discoveries; Table 2 is an excerpt. In Fig. 12-15 we present images for discoveries with projected proper lengths \( l_p \) in the ranges 5.1–4 Mpc, 4–3 Mpc, 3–2 Mpc, and 2–0.7 Mpc, respectively. For each GRG, we use the LoTSS DR2 resolution \( \theta_{\text{FWHM}} \in (6', 20', 60', 90') \) that most clearly conveys the morphology through a single image. The selection reflects our sample’s diversity in shapes and sizes and provides a sense of the data quality.

3.6.1. Angular lengths

The angular length distribution of the newly found GRGs is as follows: the smallest \( \phi = 1.5' \), the median \( \phi = 5.1' \), the largest \( \phi = 2.2' \), and 80% of angular lengths fall within [3.4', 9.8']. Thirteen of our discoveries — listed in Table 3 — are larger than the Moon in the sky (whose angular diameter varies over time, but here taken to be \( \phi = 30' \)). Our search more than doubles the known number of such spectacular GRGs — from 10 to 23.

The GRG associated with NGC 2300 (see the middle-left panel of Fig. 16) is the largest-angular–length giant ever found, and the largest-angular–length radio galaxy in the Northern Sky.\(^{33}\) It remains possible that the GRG has been generated by spiral galaxy NGC 2276 instead, with which elliptical galaxy NGC 2300 is interacting. However, this scenario seems unlikely, as only a fraction \( \sim 10^{-3} \) of known GRGs are hosted by spirals. Its discovery emphasizes that low-frequency interferometers like the LOFAR and the MWA, which are sensitive to degree-scale angular scales, are important to complete a low-redshift census of giant radio galaxies. Sky-wide, a simple extrapolation of our findings suggests that several \( \sim 10^3 \) degree-scale angular length giants similar to the GRG of NGC 2300 still await discovery at the LoTSS DR2 depth.

3.6.2. Redshifts

The redshift distribution of the newly found GRGs is as follows: the lowest \( z = 0.00635 \pm 6 \cdot 10^{-5} \), the median \( z = 0.29 \), the highest \( z = 2.6394 \pm 6 \cdot 10^{-4} \), and 80% of redshifts fall within [0.12, 0.68]. Because of our focus on GRGs of large angular length (see Sect. 3.2), we have found only 36 GRGs beyond \( z > 1 \). One of these, the GRG at rank 13 of Table E.1, is the largest secure GRG found beyond redshift 1. Its estimated distance is \( l_p = 3.9 \pm 0.1 \) Mpc, and it is the only known GRG that is still visible at the LoTSS DR2 depth.

3.6.3. Projected proper lengths

With \( l_p = 5.1 \pm 0.2 \) Mpc and \( l_p = 4.99 \pm 0.04 \) Mpc, our LoTSS DR2 sample contains the first two 5 Mpc–scale giants. We have presented a dedicated analysis of the latter GRG in Oei et al. (2022a). 11 discoveries have \( l_p > 4 \) Mpc, 53 have \( 3 \leq l_p < 4 \) Mpc, 289 have \( 2 \leq l_p < 3 \) Mpc, 1212 have \( 1 \leq l_p < 2 \) Mpc, and 485 have \( l_p < 1 \) Mpc. The median \( l_p = 1.36 \) Mpc, and 80% of projected proper lengths fall within [0.83 Mpc, 2.29 Mpc].

3.6.4. Stellar and supermassive black hole masses

Following Oei et al. (2022a), we collected host stellar masses \( M_\star \) from Chang et al. (2013) and Salter et al. (2018), and estimated host SMBH masses \( M_{\text{BH}} \) via SDSS DR12 stellar velocity dispersions (Alam et al. 2015) and the M-sigma relation of Kormendy & Ho (2013)’s Eq. (7). From all 3342 GRGs in our final catalogue, only 747 (22%) could be assigned a stellar mass in this way, and 80% of host SMBH masses fall within [1.8 \cdot 10^{10} M_\odot, 5.3 \cdot 10^{10} M_\odot]. We discover two GRGs whose hosts, J150329.07+374850.3 and J073505.24+415827.5, are the least massive known: both have a stellar mass \( M_\star = 4.8 \cdot 10^{10} M_\odot \). These are small giants, with \( l_p = 0.8 \) Mpc and \( l_p = 0.7 \) Mpc, respectively. The top panel of Fig. 8.1 hints at a weak positive correlation between \( M_\star \) and \( l_p \), which future work should confirm or reject.

The median \( M_\star = 3.4 \cdot 10^{11} M_\odot, \) and 80% of stellar masses fall within [1.8 \cdot 10^{11} M_\odot, 5.3 \cdot 10^{11} M_\odot]. We discover two GRGs whose hosts, J150329.07+374850.3 and J073505.24+415827.5, are the least massive known: both have a stellar mass \( M_\star = 4.8 \cdot 10^{10} M_\odot \). These are small giants, with \( l_p = 0.8 \) Mpc and \( l_p = 0.7 \) Mpc, respectively. The top panel of Fig. 8.1 hints at a weak positive correlation between \( M_\star \) and \( l_p \), which future work should confirm or reject.

The median \( M_\star = 1.0 \cdot 10^{9} M_\odot, \) and 80% of SMBH masses fall within [0.4 \cdot 10^{9} M_\odot, 2.3 \cdot 10^{9} M_\odot]. The SMBH masses of J123703.24+275819.5, J163659.07+541725.4, and J103129.54+502959.1 are the highest estimated yet, with

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\(^{33}\) This catalogue, complete up to and including April 2020, contains the GRGs found in 40 prior publications (for a list, see Dabhade et al. 2020a). Section 1: Introduction, alongside their own discoveries.

\(^{34}\) At \( a = 8' \), the Southern Sky’s Centaurus A (Cooper et al. 1965) is the largest-angular-length radio galaxy overall (e.g. McKinley et al. 2022), despite this, at \( l_p = 0.48 \) Mpc, it is not a giant.
Fig. 11: Mollweide view of the sky showing locations of all known giants, of which 61% are discoveries presented in this work. In the background, we show the specific intensity function of the Milky Way at ν_{obs} = 150 MHz (Zheng et al. 2017). The LoTSS DR2 has avoided the Galactic Plane, where extended emission complicates calibration and deconvolution. Our search footprint encloses a grey spherical rectangle, which represents the Boötes LOFAR Deep Field search by Simonte et al. (2022), and a grey spherical cap, which represents the Bootes LOFAR Deep Field search by Simonte et al. (2022).

\[ M_\bullet = 2 \cdot 10^{11} M_\odot, \ M_\bullet = 5 \cdot 10^{10} M_\odot, \text{ and } M_\bullet = 5 \cdot 10^{10} M_\odot, \] respectively. The latter masses equal the theoretical maximum mass of accreting black holes of typical spin (King 2016). Curiously, although J103129.54+502959.1’s \( M_\bullet \) is among the highest estimated SMBH mass of any GRG host, the GRG itself is relatively small: \( l_p = 0.81 \pm 0.06 \) Mpc. Conversely, the bottom panel of Fig. 16 shows that multi-Mpc giants can have hosts with SMBH masses that are two orders of magnitude lower than J103129.54+502959.1’s.

3.6.5. Spiral or lenticular host galaxies

Remarkably, although NGC 6185 is a spiral galaxy of Hubble-de Vaucouleurs class Sa (Jansen et al. 2000), it appears to have generated the GRG shown in the top-left panel of Fig. 16. Such systems are exceedingly rare: not only are few RGs galaxy–hosted radio galaxy. Hitherto, the largest spiral galaxy–hosted GRG in the literature has been J2345-0449 (Bagchi et al. 2014), with \( l_p = 1.6 \) Mpc. Given the favourably low redshift of \( z = 0.03430 \pm 7 \cdot 10^{-5} \), NGC 6185 and its enigmatic GRG solicit a dedicated analysis (Oei et al. in prep.). Besides NGC 6185, spiral or lenticular galaxies J080403.40+404809.9 and J091459.66+294348.8 (known alternatively as NGC 2789; see the bottom-right panel of Fig. 16) have also generated GRGs; these have projected proper lengths \( l_p = 1.1 \) Mpc and \( l_p = 0.9 \) Mpc, respectively. Morphological host classification through SDSS, Pan-STARRS, and DESI imagery is reliable only up to \( z \sim 0.1–0.2 \), depending on viewing angle and various other factors. Our LoTSS DR2 sample contains 342 GRGs with definite hosts at \( z = 0.15 \) or below, among which are all 3 GRGs with spiral or lenticular hosts discussed here. It thus appears that the fraction of GRG hosts that is of such non-elliptical nature is \( \sim 1\% \).

A more detailed morphological characterisation of the sample appears possible, for example using data from Hart et al. (2016), but this is beyond the scope of the current work.

4. Results

After first building a statistical framework and then collecting a large sample of GRGs from a single survey through a systematic approach, we are ready to infer the intrinsic GRG length distribution. In particular, we aim to establish whether the RG intrinsic proper length RV \( L \) is well described by a Pareto distribution, and if so, what its tail index \( \xi \) is. Subsequently, we infer derived quantities — such as the comoving GRG number density in the Local Universe.

4.1. Giant radio galaxy length distribution

4.1.1. Empirical survival function

From our LoTSS DR2 GRG sample, we compute the empirical cumulative distribution function (ECDF) of the GRG observed projected proper length RV \( L_{p,\text{obs}} \) \( L_{p,\text{GRG}} \) (see Eq. 21). We only include GRGs for which \( l_p \) itself — rather than a lower bound — is known, and set either \( z_{\text{max}} = 0.5 \) or \( z_{\text{max}} = 0.25 \); this retains 1462 or 800 GRGs for analysis, respectively. The empirical survival function (ESF), which equals one minus the ECDF, is shown in dark grey in both panels of Fig. 17. Just as

\[ F_L(l) = 1 - \frac{1}{\xi} \left( \frac{l}{l_0} \right)^{-\xi - 1}, \]

\[ l \geq l_0 \]

we explore two choices for \( z_{\text{max}} \), as our model assumes that \( \xi \) remains constant between \( z \in [0, z_{\text{max}}] \). We further discuss this assumption in Section 5.5.
Fig. 12: Details of the LoTSS DR2-estimated specific intensity function $I_\nu (\hat{r})$ at central observing frequency $\nu_{\text{obs}} = 144 \text{ MHz}$ and resolutions $\theta_{\text{FWHM}} \in \{6'' , 90''\}$, centred around the hosts of newly discovered GRGs. Row-wise from left to right, from top to bottom, the projected proper length $l_\parallel$ is 5.1 Mpc, 5.0 Mpc (Oei et al. [2022a]), 4.6 Mpc, 4.6 Mpc, 4.1 Mpc, and 4.1 Mpc; in the same order, $\theta_{\text{FWHM}}$ is $6''$, $90''$, $6''$, $90''$, $6''$, and $6''$. The GRG in the bottom-left panel appears larger in the sky than the Moon. In the middle-right panel, contours signify 2.5 and 3.5 sigma-clipped standard deviations (SDs) above the sigma-clipped median; in the bottom-right panel, they signify 3, 5, and 10 such SDs. For scale, we show the stellar Milky Way disk (with a diameter of 50 kpc) generated using the Ringermacher & Mead [2009] formula, alongside a 10 times inflated version.
Fig. 13: Details of the LoTSS DR2–estimated specific intensity function $I_r (\hat{r})$ at central observing frequency $\nu_{\text{obs}} = 144 \text{ MHz}$ and resolutions $\theta_{\text{FWHM}} \in \{6'' , 20'' , 90''\}$, centred around the hosts of newly discovered GRGs. Row-wise from left to right, from top to bottom, the projected proper length $l_p$ is 3.9 Mpc, 3.5 Mpc, 3.3 Mpc, 3.3 Mpc, 3.3 Mpc, and 3.2 Mpc; in the same order, $\theta_{\text{FWHM}}$ is $90''$, $20''$, $20''$, $6''$, $6''$, and $20''$. The GRGs in the top-left and middle-left panels appear larger in the sky than the Moon. Contours signify 3, 5 and 10 sigma-clipped standard deviations above the sigma-clipped median. For scale, we show the stellar Milky Way disk (with a diameter of 50 kpc) generated using the Ringermacher & Mead (2009) formula, alongside a 5 or 10 times inflated version.
Fig. 14: Details of the LoTSS DR2–estimated specific intensity function $I_\nu (\hat{r})$ at central observing frequency $\nu_{\text{obs}} = 144$ MHz and resolutions $\theta_{\text{FWHM}} \in \{6'' , 20''\}$, centred around the hosts of newly discovered GRGs. Row-wise from left to right, from top to bottom, the projected proper length $l_p$ is 2.8 Mpc, 2.6 Mpc, 2.6 Mpc, 2.2 Mpc, 2.1 Mpc, and 2.1 Mpc; in the same order, $\theta_{\text{FWHM}}$ is 6'', 20'', 6'', 20'', 20'', 20'', and 6''. The GRG in the bottom-left panel appears larger in the sky than the Moon. Contours signify 3, 5, and 10 sigma-clipped standard deviations above the sigma-clipped median. For scale, we show the stellar Milky Way disk (with a diameter of 50 kpc) generated using the Riemercher & Mead (2009) formula, alongside a 5 or 10 times inflated version.
Fig. 15: Details of the LoTSS DR2–estimated specific intensity function $I_\nu(\tilde{r})$ at central observing frequency $\nu_{\mathrm{obs}} = 144$ MHz and resolutions $\theta_{\mathrm{FWHM}} \in [6'' ~ 20'']$, centred around the hosts of newly discovered GRGs. Row-wise from left to right, from top to bottom, the projected proper length $l_0$ is 1.6 Mpc, 1.5 Mpc, 1.3 Mpc, 1.2 Mpc, 1.1 Mpc, and 0.7 Mpc; in the same order, $\theta_{\mathrm{FWHM}}$ is 6'', 6'', 20'', 6'', 6'' and 6''. Contours signify 3, 5, and 10 sigma-clipped standard deviations above the sigma-clipped median. For scale, we show the stellar Milky Way disk (with a diameter of 50 kpc) generated using the Ringeracher & Mead (2009) formula, alongside a 3 or 5 times inflated version.
Fig. 16: Details of the LoTSS DR2–estimated specific intensity function $I_\nu(\vec{r})$ at central observing frequency $\nu_{\text{obs}} = 144$ MHz and resolutions $\theta_{\text{FWHM}} \in \{6^\prime, 20^\prime, 90^\prime\}$, centred around the hosts of newly discovered GRGs. Row-wise from left to right, from top to bottom, the projected proper length $l_p$ is 2.5 Mpc, 1.9 Mpc, 1.1 Mpc, 1.0 Mpc, 0.9 Mpc, and 0.9 Mpc; in the same order, $\theta_{\text{FWHM}}$ is $90^\prime$, $20^\prime$, $90^\prime$, $20^\prime$, $6^\prime$, and $20^\prime$. All appear larger in the sky than the Moon. The top-left panel shows the GRG of NGC 6185, a spiral galaxy. This is the first spiral galaxy–hosted GRG known with $l_p > 2$ Mpc. The middle-left panel shows a structure we interpret as a radio galaxy belonging to the elliptical galaxy NGC 2300. At $\phi = 2.2^\circ$, this GRG has the largest angular length of all uncovered thus far.
the ECDF approximates the CDF, the ESF approximates the survival function (SF). In this case, for any \( l_p \), the ESF provides the probability that a randomly drawn LoTSS DR2 GRG will have a projected proper length exceeding \( l_p \). If \( L_{p, \text{obs}} | L_{p, \text{GRG}} > l_p \) were Parerian, its ESF (hereafter: 'the' ESF) would resemble a straight line in Fig. 17's right panel — both axes have logarithmic scaling. However, the ESF clearly displays curvature.

4.1.2. Selection effects

Under the ansatz for \( L \) is Parerian, as proposed in Sect. 4.1, the aforementioned ESF’s curvature implies a significant role for observational selection effects. The reason is the following. The ansatz implies that the GRG projected proper length RV \( L_p | L_p > l_p, \text{GRG} \) is also Parerian (see Eq. 5), as is \( L_{p, \text{obs}} | L_{p, \text{obs}} > l_p, \text{GRG} \) when selection effects are negligible (see Sect. 2.8.1) and set \( C(l_p) = 1 \). But in our case \( L_{p, \text{obs}} | L_{p, \text{obs}} > l_p, \text{GRG} \) is not Parerian: its ESF is curved. To avoid contradiction, we must relax at least one assumption. If the Pareto ansatz is held, then selection effects must be at play.

When selection effects are non-negligible, we must devise a procedure to disentangle them from the data if our \( \xi \) estimate is to be uncontaminated. To this end, we perform joint Bayesian inference with a model that includes both \( \xi \) and parameters that describe the selection effects.

In particular, we consider the roles of the fuzzy angular length threshold and surface brightness selection effects, introduced in Sects. 2.8.2 and 2.8.3 respectively. In Sect. 3.2 we explain that we have attempted to maintain a 5" angular length threshold during our LoTSS DR2 GRG search. If we want to use Sect. 2.8.2 to correct for the bias against faraway and physically small giants that this threshold has imprinted onto our sample, we must estimate parameters \( \phi_{\text{min}} \) and \( \phi_{\text{max}} \). A natural choice is to assume that they lie symmetrically around the intended angular length threshold of 5" — but at what distance from it? We propose to consider this distance, \( \frac{1}{2} (\phi_{\text{max}} - \phi_{\text{min}}) \), as a yet unknown model parameter that we fit to the data. Similarly, if we want to use Sect. 2.8.3 to correct for the bias against physically large giants that the limited depth of the LoTSS DR2 imprints onto our sample, we must estimate parameters \( b_{\text{ref}} \) and \( \sigma_{\text{ref}} \). Our approach is to, again, fit these parameters — possibly making use of any available prior knowledge.

The meaning of \( b_{\text{ref}} \) depends on the choice of \( l_{\text{ref}} \) and \( \nu_{\text{obs}} \); we define \( l_{\text{ref}} := 0.7 \text{ Mpc} \) and \( \nu_{\text{obs}} := 144 \text{ MHz} \). We furthermore set \( \nu_{\text{ref}} := 1 - \nu_{\text{obs}}, \) with \( \sigma_{\text{ref}} = 25 \text{ Jy deg}^{-2} \) being the typical LoTSS DR2 6" RMS noise (Shimwell et al. 2022).

4.1.3. Prior

We exploit two sources of prior knowledge. Firstly, we attempt to directly estimate \( b_{\text{ref}} \) and \( \sigma_{\text{ref}} \) by selecting from all LoTSS DR2 GRGs with \( l_p \leq 1 \text{ Mpc} \) a random subset of size 50 (10%). For these giants, we estimate the mean surface brightnesses of both lobes from the LoTSS DR2 6" imagery, differentiating between the brighter and the fainter lobe. Because our goal is to estimate \( b_{\text{ref}} \), we attempt to undo cosmological and growth-induced surface brightness dimming assuming a universal lobe spectral index \( \alpha = -1 \) and self-similar growth: \( \nu = \frac{2}{3} \). The resulting surface brightnesses correspond to \( z = 0 \), and to the epoch in each giant’s life when \( l = l_{\text{ref}} \). The median of the corrected bright lobe mean surface brightnesses is \( b_{\text{ref}} = 1.3 \times 10^3 \text{ Jy deg}^{-2} \), whilst the median of the corrected faint lobe mean surface brightnesses is...
\[ b_{v,\text{ref}} = 0.7 \cdot 10^3 \text{ Jy deg}^{-2}. \] All lobes taken together, the median becomes \( b_{v,\text{ref}} = 1.0 \cdot 10^3 \text{ Jy deg}^{-2}. \) We perform maximum likelihood estimation assuming the surface brightness distribution is lognormal and find \( \sigma_{\text{ref}} = 1.3, \) again using all lobes. Note, however, that we have only used observed giants here, whilst \( b_{v,\text{ref}} \) and \( \sigma_{\text{ref}} \) should correspond to the entire population of giants. As fainter giants will have preferentially fallen out, we might have overestimated \( b_{v,\text{ref}} \) and underestimated \( \sigma_{\text{ref}}. \)

To probe whether we have overestimated \( b_{v,\text{ref}} \) and underestimated \( \sigma_{\text{ref}} \), we use data from Dabhade et al. (2020b) and a Monte Carlo approach. First, for their sample of 239 LoTSS DR1 GRGs, we compute total luminosity densities \( L_{\nu} \) at rest-frame frequency \( \nu = 144 \text{ MHz}. \) (Given that LoTSS DR1 and DR2 noise levels are similar, this population will also be representative of the LoTSS DR2.) In the top panel of Fig. 18, we show \( L_{\nu} \) versus \( l_p \) for all 139 for which \( z < 0.6. \) To increase the range of lengths covered, we additionally show data on Alcyoneus (Oei et al. 2022a). Next, under the same assumptions of a constant spectral index and self-similar growth, we derive a simple luminosity density–surface brightness relationship that allows for back-and-forth conversion between the two — at least, given projected lengths and redshifts. The mean lobe surface brightness \( \langle b_v \rangle \) is proportional to the total luminosity density \( L_{\nu} \), and assuming a pair of spherical lobes

\[ \langle b_v \rangle = \frac{2 f_{L\nu} \cdot L_{\nu}}{\pi^2 \cdot \mathcal{E}[D](\eta(f_l)) \cdot f_l^2 \cdot f_l^2 \cdot (1 + z)^{3 - \alpha}}. \]  

(40)

Here \( f_{L\nu} \) is the fraction of the total luminosity density that belongs to the lobes, \( f_l \) is the fraction of the RG's axis length that lies inside the lobes, and \( \mathcal{E}[D](\eta(f_l)) \) is the mean deflection factor as given by Eq. (4.29). The peak surface brightness \( b_{v,\text{max}} \) relates to \( \langle b_v \rangle \) as \( b_{v,\text{max}} = \langle b_v \rangle \). Section C contains derivations for both these results. For Alcyoneus, at \( z_{\text{Alc}} \approx 0.25 \), \( f_{L\nu} = 0.3 \) and \( f_l = 0.3 \) (Oei et al. 2022a). Assuming these parameter values, again in the top panel of Fig. 18, we show luminosity density–projected length pairs of RGs at \( z = z_{\text{Alc}} \). whose peak (solid green curve) or mean (dashed-dotted green curve) surface brightness equals the LoTSS DR2 noise level. Thus, each curve represents a family of borderline-detectable giants at Alcyoneus’s redshift. The other dash-dotted curves indicate similar barely detectable families, but for other redshifts. Without optimising any free parameters, the curves correctly predict that Alcyoneus’s lobes have surface brightnesses comparable to the 6σ LoTSS DR2 noise level and explain the absence of observations in the top-left corner of the figure. We conclude that Eq. (40) appears reasonable, but note that RGs may significantly differ in their values of \( f_{L\nu} \) and \( f_l \). Bolstering, we make use of Eq. (40) to Monte Carlo simulate — for particular values of \( b_{v,\text{ref}} \) and \( \sigma_{\text{ref}} \) — luminosity density–projected length relationships as they appear to observers. The simulated GRGs have projected lengths adopted from the observed GRGs, randomly sampled redshifts up to \( z = 0.6 \) assuming a spatially constant GRG number density, and randomly sampled reference surface brightnesses (i.e. those for RGs at \( z = 0 \) that have intrinsic length \( l = l_{\text{ref}} \)) whose distribution is determined by \( b_{v,\text{ref}} \) and \( \sigma_{\text{ref}} \). We then use Eq. (24) to compute surface brightnesses as they would be observed, and retain only those GRGs whose surface brightness exceeds the LoTSS DR2 noise level. For these simulated detectable GRGs, we finally generate luminosity densities using Eq. (40) assuming wide uniform distributions \( f_{L\nu} \sim f_l \sim \text{Uniform}(0.1, 0.9) \). The middle and bot-

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**Fig. 18:** Comparison between luminosity density–projected proper length relations for observed and simulated GRGs. Each dash-dotted curve denotes a family of giants at a given redshift, assuming \( f_{L\nu} = 0.3 \) and \( f_l = 0.3 \), whose mean lobe surface brightnesses equal the LoTSS DR2 noise level and who are thus borderline-detectable. The grey band denotes the median-centred luminosity density range that contains 60% of GRGs. **Top:** 139 LoTSS DR1 GRGs (Dabhade et al. 2020b), alongside Alcyoneus, with redshift \( z_{\text{Alc}} \approx 0.25 \) (Oei et al. 2022a). The solid green curve is similar to the dash-dotted green curve, but represents maximum instead of mean lobe surface brightness. **Middle:** 1000 simulated GRGs, assuming \( b_{v,\text{ref}} = 1750 \text{ Jy deg}^{-2} \) and \( \sigma_{\text{ref}} = 1.3. **Bottom:** 1000 simulated GRGs, assuming \( b_{v,\text{ref}} = 250 \text{ Jy deg}^{-2} \) and \( \sigma_{\text{ref}} = 1.3. **---**
tom panels of Fig. 18 show results for \( b_{\text{ref}} = 1750 \) Jy deg\(^{-2}\) and \( b_{\text{ref}} = 250 \) Jy deg\(^{-2}\), respectively; we adopt \( \sigma_{\text{ref}} = 1.3 \) from our LoTSS DR2 GRG surface brightness measurements. The median luminosity density of the \( z < 0.6 \) LoTSS DR1 GRGs is \( L_{\text{z}} = 1.1 \cdot 10^{26} \) W Hz\(^{-1}\), that of the \( b_{\text{ref}} = 1750 \) Jy deg\(^{-2}\) simulated GRGs is \( L_{\text{z}} = 0.6 \cdot 10^{26} \) W Hz\(^{-1}\), and that of the \( b_{\text{ref}} = 250 \) Jy deg\(^{-2}\) simulated GRGs is \( L_{\text{z}} = 0.2 \cdot 10^{26} \) W Hz\(^{-1}\) \[^{17}\]

Interestingly, the higher reference surface brightness median provides a better fit to the data. Even in case our Monte Carlo approach would predict luminosity densities that are biased low by a factor two, the higher median remains favoured.

In conclusion, it seems reasonable to suppose that our measurement \( b_{\text{ref}} = 1.0 \cdot 10^{3} \) Jy deg\(^{-2}\) is not, or only mildly, biased high by selection effects. Still, we take a conservative approach and in setting priors we assume a 75% error on our measurement of \( b_{\text{ref}} \) and a 50% error on our measurement of \( \sigma_{\text{ref}} \). Thus, the priors for \( b_{\text{ref}} \) and \( \sigma_{\text{ref}} \) — which we choose to be Gaussian — have 68% credible intervals \([250 \text{ Jy deg}^{-2}, 1750 \text{ Jy deg}^{-2}]\) and \([0.65, 1.95]\). We retain flat priors for \( \xi \) and \( \frac{1}{2} (\phi_{\max} - \phi_{\min}) \).

4.1.4. Inference

To compute the posterior distribution for \( \xi \), \( \frac{1}{2} (\phi_{\max} - \phi_{\min}) \), \( b_{\text{ref}} \) and \( \sigma_{\text{ref}} \), we first brute-force evaluate the likelihood function over a regular grid that covers a total of 3.3 million parameter combinations. For each proposed parameter quartet, we compute the PDF of \( \omega_{\text{obs}} \mid L_{\text{obs}} > L_{\text{GRG}} \) and obtain the likelihood assuming that the LoTSS DR2 GRG projected proper lengths are IID draws from it. To obtain the PDF, we successively evaluate Eqs. 22, 25, 29, 18, 3, and 21 alongside their direct dependencies. This requires the numerical evaluation of five integrals. Compared to using Riemann sums, we achieved substantial accuracy improvements at virtually no added numerical cost by approximating these integrals with the trapezoid rule and the composite Simpson’s rule.

We summarise the likelihood function in Table 4.1 and Fig. 4.1. To obtain the posterior, we simply multiply the likelihood function by the prior and normalise the result.

Table 4: Maximum a posteriori probability (MAP) and posterior mean and standard deviation (SD) estimates of the free parameters in intrinsic GRG length distribution inference. The model assumes \( \xi \) is constant for \( z \in [0, \zeta_{\text{max}}] \). We determine the posterior twice: for \( \zeta_{\text{max}} = 0.5 \), using 1462 GRGs, and for \( \zeta_{\text{max}} = 0.25 \), using 800 GRGs.

\[
\begin{array}{ccc}
\text{parameter} & \text{MAP} & \text{posterior mean and SD} \\
\xi & -3.55 & -3.5 \pm 0.5 \\
\frac{1}{2} (\phi_{\max} - \phi_{\min}) & 1.9' & 1.9 \pm 0.2' \\
b_{\text{ref}} & 600 \text{ Jy deg}^{-2} & 720 \pm 380 \text{ Jy deg}^{-2} \\
\sigma_{\text{ref}} & 1.2 & 1.2 \pm 0.2 \\
\zeta_{\text{max}} & 0.5: \\
\end{array}
\]

In Table 4 for each parameter, we list the maximum a posteriori probability (MAP) estimate, alongside estimates for the posterior mean and standard deviation. In Fig. 19 we visualise all one- and two-dimensional posterior marginals, in which we mark the MAP (white circle) and the posterior mean (white cross). The joint marginal for \( \xi \) and \( b_{\text{ref}} \) shows that these parameters have a strong negative correlation, indicating that with current data, the steep slope of the ESF at high \( \ell_{p} \) can equally be described with a steep intrinsic slope and mild surface brightness selection (i.e. \( \xi \) low and \( b_{\text{ref}} \) high), or by a shallow intrinsic slope and strong surface brightness selection (i.e. \( \xi \) high and \( b_{\text{ref}} \) low). We leave it up to future studies to break this degeneracy, either by using larger samples, by measuring \( b_{\text{ref}} \) directly, or by improving survey sensitivities so that surface brightness selection effect modelling becomes superfluous altogether.

4.1.5. Goodness of fit

In both panels of Fig. 17 we compare the ESF and SF of \( L_{\text{obs}} \mid L_{\text{obs}} > L_{\text{GRG}} \) for \( \zeta_{\text{max}} = 0.5 \), using the MAP parameters for the latter. The model appears able to produce a tight fit to the data. The mean and standard deviation of the ESF–SF residuals are 0.01% and 0.3%, whilst the mean and standard deviation of the absolute ESF–SF residuals are 0.2% and 0.2%. Using a Kolmogorov–Smirnov test, we formally verify that our best parameters are indeed good parameters — in the sense that they represent a plausible model underlying the data. The Kolmogorov–Smirnov statistic is the maximum deviation between the ESF and SF, and equals 1% in our case. The \( p \)-value — the probability that an ESF–SF discrepancy of at least this magnitude would occur if the SF represents the true underlying distribution — is \( p = 99\% \). For any reasonable significance level, we do not reject the null hypothesis. The model, given our best parameters, indeed represents a possible description of the data. We conclude that the distribution of GRG intrinsic proper lengths, after correcting for selection effects, is consistent with a single Pareto distribution with tail index \( \xi = -3.5 \). We show the SF of this distribution in both panels of Fig. 17 (fading green dots). For low \( \ell_{p} \), the observed slope is shallower (due to angular length selection), whilst for high \( \ell_{p} \), the observed slope is steeper (due to surface brightness selection).

A quasi-Pareto distribution can arise naturally as the tail of a lognormal distribution (e.g. \citealp{Malevergne2011}), and there are reasons to believe that the entire radio galaxy length distribution is indeed approximately lognormal (\citealp{Oei2021}). This provides an explanation of the approximately Pareto nature of the giant radio galaxy length distribution found in this section. The specific value of the tail index \( \xi \) is set by both the physics of radio galaxy growth and the distribution of radio galaxies over large-scale environments, the latter of which we measure in \citealp{Oei2021}. Our result \( \xi = -3.5 \pm 0.5 \) is a new constraint for dynamical models such as those of \cite{Turner2015} and \cite{Hardcastle2018}.

18 For another interesting case, \( b_{\text{ref}} = 500 \) Jy deg\(^{-2}\) (not shown), the median \( L_{\text{z}} = 0.3 \cdot 10^{26} \) W Hz\(^{-1}\).
19 The computation takes a few thousand CPU hours to complete, but can be trivially distributed among nodes, and within a node among CPUs. Model extensions that introduce additional parameters necessitate more efficient inference techniques, such as Markov chain Monte Carlo.

Article number, page 22 of 68
Fig. 19: With 1462 projected lengths of LoTSS DR2 giants up to $z_{\text{max}} = 0.5$, we estimate the joint posterior distribution over $\xi$ — the parameter of interest — and $\frac{1}{2}(\phi_{\text{max}} - \phi_{\text{min}})$, $b_{\nu,\text{ref}}$ and $\sigma_{\text{ref}}$ — the selection effect parameters. We show all two-parameter marginals of the posterior, with contours enclosing 30% and 70% of total probability. We mark the maximum a posteriori probability parameters (white circle) and the posterior mean parameters (white cross). The single-parameter marginals again show the estimated posterior mean, now marked by a vertical line, alongside shaded median-centred 80% credible intervals. To compare the posterior to the likelihood function, which is also the posterior for a uniform prior, see Fig. D.1.

4.2. Giant radio galaxy number density

If in addition to our discoveries, we know how many giants our LoTSS DR2 search campaign has missed, then we can infer the true comoving GRG number density in the Local Universe. The posterior distribution over selection effect parameters $\frac{1}{2}(\phi_{\text{max}} - \phi_{\text{min}})$, $b_{\nu,\text{ref}}$ and $\sigma_{\text{ref}}$ induces a probability distribution over the search completeness function $C(l_p)$. $C(l_p)$ denotes the probability that a giant of projected proper length $l_p$ in comoving space up to $z = z_{\text{max}}$ is detected through the search. We first generate parameter samples from our posterior using rejection sampling, and then use each to calculate a $C(l_p)$ sample. We show the distribution over $C(l_p)$ for $z_{\text{max}} = 0.5$ in Fig. 20. For small $l_p$, $C$ is low as many GRGs drop out due to angular length selection; for large $l_p$, $C$ is low as many GRGs drop
For comparison, in a (100 Mpc)$^3$ volume up to $z_{\text{max}} = 0.5$, the SDSS-III cluster catalogue of [12] contains on average 11.2 clusters of any mass, and 4.5 clusters of mass $M_{200} > 10^{12} M_{\odot}$. Since clusters contain ~20% of GRGs ([28]), we find the average number of GRGs per cluster to be $\sim 10^{-3}$. If one assumes that filaments contain the remaining ~80% of GRGs, and uses the fact that the average number of filaments per cluster is of order unity, it follows that the average number of giants per filament is also $\sim 10^{-4}$. In all likelihood, most clusters and filaments do not currently contain a giant.

4.3. Giant radio galaxy lobe volume-filling fraction

Because giant radio galaxies enrich the IGM with hot plasma and magnetic fields far beyond the circumgalactic media of their hosts, they may provide a meaningful contribution to the heating and magnetisation of—in particular—the most rarefied parts of the filament IGM. By combining the GRG number density and the GRG jet power distribution (e.g. [29], one could estimate the instantaneous heating and magnetisation contributions directly. We recommend such analysis for future research.

We evaluate Eq. [35] to obtain an estimate of the fraction of the Local Universe’s proper volume that GRG lobes occupy. We use Alcyoneus as a reference giant, for which $V = 2.5 \pm 0.3$ Mpc$^3$ and $l_p = 4.99 \pm 0.04$ Mpc ([28]); this suggests $E[V_{\text{f}}] \approx 2\%$. Future work should determine whether Alcyoneus’s case is typical, as observations, such as those shown in Figs. 12–16, suggest that giants exhibit a large variety of shapes—and thus total lobe volume—cubed length ratios. Interestingly, simulations by [31] have found that these shapes also depend on environmental parameters such as ambient pressure and density. Truncating the GRG projected length distribution at $l_{\text{max}} = 7$ Mpc, so that its support is exactly an order of magnitude, Eq. [35] predicts $V_{\text{f}}(z = 0) = 3.4 \cdot 10^{-7}$.

Whether this result is sensitive to changes in $l_{\text{max}}$ depends on $E$, with $E = -3$ being a special value under self-similar growth. In that case, small and large giants dominate equally to $V_{\text{f}}$: although large giants are rarer ($l_p \propto l_p^3$), their larger lobe volumes ($V \propto l_p^3$) exactly compensate. For $E < -3$, small giants provide the dominant contribution to $V_{\text{f}}$ and the choice of $l_{\text{max}}$ can be irrelevant; for $E > -3$, large giants dominate and the choice of $l_{\text{max}}$ always matters.

If we assume that giants occur in clusters and filaments only and use the fact that clusters and filaments comprise about 5% of the Local Universe’s volume ([32], then the GRG lobe VFF within clusters and filaments specifically is $V_{\text{f}}(z = 0) = 5.4 \cdot 10^{-6}$. We conclude that, at each given moment, GRG lobes occupy just a small fraction of the WHIM and ICM. If the enrichment of the IGM by giants is to affect the WHIM and ICM on a large scale, mixing processes in the IGM are necessary and many galaxies must be able to form giants at some point in their evolution.

4.4. Unification model constraints from quasar and non-quasar giants

In Sect. 2.11, we have predicted general ramifications of the basic unification model on a GRG sample. Here we constrain and test this model with our LoTSS DR2 GRG sample.

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**Fig. 20:** Completeness $C$ of a sample of giant radio galaxies up to cosmological redshift $z_{\text{max}} = 0.5$ is a function of projected proper length $l_p$. From samples of the posterior distribution, we infer the LoTSS DR2 GRG search campaign completeness up to $z_{\text{max}} = 0.5$. We show completeness curves for five randomly selected samples (grey) and for the posterior mean (dark green). We also show an interval around the completeness mean with the completeness standard deviation (SD) as the half-width (light green). The completeness peaks around $l_p = 2$ Mpc.

**Fig. 21:** In the Local Universe, the average number of giant radio galaxies per comoving cube with 100 Mpc sides is $4.6 \pm 2.4$. We define GRGs through $l_{\text{GRG}} = 0.7$ Mpc, and define the Local Universe to be up to cosmological redshift $z_{\text{max}} = 0.5$. We show the mean alongside a shaded median-centred 80% credible interval.

out due to surface brightness selection. The completeness peaks around $l_p \sim 2$ Mpc; however, even there the majority of GRGs remains undetected.

We infer a probability distribution over the true comoving GRG number density $n_{\text{GRG}}$ by combining Eqs. [15] and [19] with the LoTSS DR2 GRG catalogue and samples from Sect. 4.1.1’s posterior. The resulting skewed distribution, with mean and SD $n_{\text{GRG}} = 4.6 \pm 2.4$ ($100$ Mpc)$^{-3}$ and $80\%$ credible interval $3.1$–$6.7$ ($100$ Mpc)$^{-3}$, is shown in Fig. 21. Note that, although the uncertainty in $n_{\text{GRG}}$ induces a large uncertainty in $C$ from $l_p \sim 1.5$ Mpc onwards, the completeness uncertainty at large projected lengths does not substantially contribute to the uncertainty in $n_{\text{GRG}}$. This is because the GRG population is dominated by smaller giants, for which the completeness appears better constrained.

What picture arises regarding the abundance of giant radio galaxies in the Local Universe’s large-scale structure? If we model the Cosmic Web through comoving cubic unit cells ([28]) with 50 Mpc sides, and each cubic unit cell contributes one cluster and three filaments, then a cube with 100 Mpc sides features a total of eight clusters and 24 filaments.
GRGs (22%) (of which 33 (13%) are discoveries presented in this work) and 917 are non-quasar GRGs (78%) (of which 713 (78%) are discoveries presented in this work). Therefore, the apparent LoTSS DR2 quasar GRG fraction $f_Q = \frac{33}{124} = 4\%$. However, spectral class labels are preferentially available for GRG hosts with higher optical flux densities, such as those at low redshifts or those containing quasars, because they are more probable spectroscopic targets. Through Fig. 22 we demonstrate that the fraction of observed GRG hosts with spectral class labels indeed decreases with redshift, whilst the fraction of quasar identifications increases. For each redshift interval, in white, we denote the fraction of quasar GRGs within the classified population. If spectral class labels would have been available for the non-classified observed populations as well, the quasar GRG fractions would probably have been lower. Assuming that all observed GRGs whose hosts have an unknown spectral class are non-quasar GRGs, we find quasar GRG fractions $f_Q = 3\%$, 4\%, 7\%, 14\%, and 26\%, for redshift intervals 0–0.2, 0.2–0.4, 0.4–0.6, 0.6–0.8, and 0.8–1, respectively. The true quasar GRG fraction for a given redshift interval might still differ from the aforementioned observed quasar GRG fraction: namely, if selection effects make a given quasar GRG easier (or harder) to detect than a given non-quasar GRG. At higher redshifts, quasar GRGs certainly appear easier to detect than non-quasar GRGs, as hosts without quasars often become too faint to optically identify. For the lowest redshift intervals, this problem does not exist, and we therefore consider the observed quasar GRG fractions $f_Q = 3\%–4\%$ to be closest to the true one. A thorough analysis of the impact of selection effects on $f_Q$ is topic for future research. Using Eq. 37 and assuming a quasar GRG probability $p_Q > 4\%$, we find a maximum inclination angle $\theta_{\text{max}} = 37_2^{+3}_3$. For $p_Q \sim\text{Uniform}(3\%, 5\%)$, the result remains the same. In conclusion, if the basic unification model considered in Sect. 2.11 is correct, then observations of GRGs predict that quasars are AGN seen along lines-of-sight that make an angle of at most $\theta_{\text{max}} = 37_2^{+3}_3$ with the black hole rotation axis. Finally, we test whether the RV $l_{p,\text{obs}} | l_{p,\text{obs}} \geq l_{p,\text{GRG}}$ has the same distribution for quasar and non-quasar GRGs, as predicted by the unification model. In the top panel of Fig. 23 we show PDFs approximated through kernel density estimation (KDE) for newly discovered (LoTSS DR2) SDSS-classified quasar GRGs and non-quasar GRGs up to $z_{\text{max}} = \infty$. Despite the small number of quasar GRGs, $N_Q = 33$, the two-sample Kolmogorov–Smirnov (KS) test yields a low p-value $p < 1\%$. We reject the null hypothesis that these distributions stem from a single underlining one. However, this does not mean that the unification model hypothesis should immediately be rejected as well, because the selective availability of SDSS DR12 spectral class labels induces a severe selection effect. This effect can be tempered by choosing a lower $z_{\text{max}}$, but for choices such as $z_{\text{max}}$.

In fact, the top panel of Fig. 23 shows expected behaviour for a GRG search campaign with a (fuzzy) angular length threshold selection effect if the unification model is correct. At high redshifts, a GRG must have a larger projected length to pass the angular length threshold than at low redshifts. Thus, sampled high-redshift GRGs are physically larger than sampled low-redshift GRGs. Because the fraction of high-redshift quasar GRGs that is detectable and spectrally classifiable is higher than the fraction of high-redshift non-quasar GRGs that is detectable and spectrally classifiable, sampled quasar GRGs will be physically larger than sampled non-quasar GRGs. To draw this latter conclusion, we must also invoke the fact that, under the unification model, the projected length distributions of quasar and non-quasar GRGs are the same.

First, for all 3135 GRGs with definitively identified hosts, we query the SDSS DR12 spectral class $spCl$: a Boolean label indicating whether or not the host contains a quasar. As many hosts have no SDSS DR12 spectrum, or even fall outside of SDSS DR12 coverage, we retrieve host classifications for just 1177 of these GRGs (38\%). Of these classified GRGs, 260 are quasar GRGs, 33 are discovery quasar GRGs, and 917 are all non-quasar GRGs. Of these classiﬁed GRGs, 260 are quasar GRGs (38\%), Of these classiﬁed GRGs, 260 are quasar GRGs (38\%). Of these classiﬁed GRGs, 260 are quasar GRGs (38\%). Of these classiﬁed GRGs, 260 are quasar GRGs (38\%). Of these classiﬁed GRGs, 260 are quasar GRGs (38\%). Of these classiﬁed GRGs, 260 are quasar GRGs (38\%).
\[ z_{\text{max}} = 0.2, 0.4, \text{or} 0.5, \text{just} N_Q = 3, 8, \text{and} 9 \text{ quasar GRGs remain} \]
\[ \text{too few to extract meaningful information. The bottom panel of Fig. 23 again shows observed proper length KDE PFDs, but now for all SDSS-classified quasar GRGs and non-quasar GRGs up to } z_{\text{max}} = \infty; \text{in this case, } N_Q = 260. \text{Again, the}\]
\[ \text{two-sample KS test yields } p < 1\%, \text{but this time quasar GRGs appear smaller than non-quasar GRGs. Because we now aggregate samples that have different selection effects imprinted, it}\]
\[ \text{becomes hard to draw clear conclusions. Limiting } z_{\text{max}} \text{reduces the severity of most selection effects, but of course comes at the cost of reducing the sample size. Interestingly, the corresponding quasar GRG and non-quasar GRG projected length distributions do become more alike; e.g. for } z_{\text{max}} = 0.5 \text{ and } 0.4, \text{the } \]
\[ \text{two-sample KS test yields } p = 2\% \text{ and } 28\%, \text{respectively. In conclusion, because quasar GRGs are intrinsically rare and the availability of spectral class labels is biased towards low redshifts and hosts containing quasars, it is challenging to robustly test the unification model with current GRG observations. We refrain from drawing final conclusions, and recommend a careful future analysis.}\]

5. Discussion

5.1. Radio galaxy length definitions

How large are radio galaxies? Despite the simplicity of this question and more than half a century of research on radio galaxies, their intrinsic length distribution has not yet been rigorously characterised. In this work, we have carried out the first precision analysis of the tail of the radio galaxy intrinsic length distribution. Precision analyses tend to raise questions; firstly, whether the studied observable is well defined, and secondly whether one could conceive of more informative observables — i.e. those that make it easier to reveal underlying physical mechanisms. This work’s main observable is the radio galaxy projected proper length; we argue that it is neither well defined nor maximally informative.

5.1.1. The current length definition: survey-dependence

Contemporary research uses a survey-dependent definition for radio galaxy angular lengths, which then makes projected proper lengths survey-dependent too. The angular length is canonically defined as the largest possible angular separation between two directions for which the RG’s specific intensity function \( I_{\nu,\text{RG}} \) exceeds \( b_{\nu,\text{th}} \): some specified factor of order unity times the image noise \( \sigma I_{\nu,\text{th}} \). A complication is that not only \( I_{\nu,\text{RG}} \), but also \( \sigma I_{\nu,\text{th}} \), varies with observing frequency; the latter because of observational factors such as (a) \( v \nu \)-coverage, bandpass, radio-frequency interference (RFI), ionospheric weather, the sky density of bright calibrators and the performance of calibration algorithms. \( I_{\nu,\text{RG}} \) additionally depends on resolution, at least for point sources; \( \sigma I_{\nu,\text{th}} \) additionally depends on resolution and integration time. As a result, both \( I_{\nu,\text{RG}} \) and the concrete value of \( b_{\nu,\text{th}} \) used in the angular length definition change from image to image. Each study thus far has therefore implicitly used a different definition for angular length, instead of a shared, absolute one. As the projected proper length follows from combining the angular length with the host redshift, it suffers from the same problem.

Whether the survey-dependence of the current length definition is problematic, depends on the radio galaxy. For archetypal FRII RGs, the angular length is roughly equal to the angular distance between the hotspots. These are an FRII RG’s brightest morphological components, and are thus the first to be picked up by a survey. In contrast, archetypal FRI RGs, which gradually fade with distance from the host, can have significantly larger angular lengths in surveys of higher sensitivity. The GRGs in the middle-right and bottom-left panel of Fig. 16 are good examples: these radio galaxies were known before the LoTSS DR2, but were not known to be giants; similarly, more sensitive surveys are poised to assign them even larger extents. If we are to move towards precision science, it therefore makes sense — at least for FRI RGs — to more explicitly recognise that the angular length is problematic, depends on the radio galaxy. For archetypal FRII RGs, the angular length is largely equal to the angular length measurement. In future images, some of the visible plasma might be of such low pressure that it has become physically insignificant, in the sense that it does not affect the thermodynamics of the surrounding IGM anymore; we propose to exclude such plasma from a radio galaxy length.

Practically, one option is to introduce an absolute threshold: e.g. to include plasma of pressure \( P \geq 10^{-17} \text{ Pa} \) only — this is the pressure of the warm-hot intergalactic medium (WHIM) in the \( \rho_{\text{WHIM}} \sim 10 \Omega_{\text{M,0}} \rho_c \) K regime. Another option is to introduce an environment-dependent pressure threshold; this would mean that a cluster RG sees its length measured against a higher pressure threshold than a filament RG, because its plasma becomes thermodynamically irrelevant sooner. A problem with (equipartition or minimum energy) pressure-based length definitions is that pressure is a derived quantity: the specific intensity only determines the product of pressure and line-of-sight length through the lobe.

\[ \text{Alcyoneus, shown in Fig. 12’s top-right panel, is a 5 Mpc giant whose ageing lobes are revealed for the first time by the LoTSS DR2. (Oei et al. 2022a). Future image sensitivity improvements shall reveal more hitherto unseen, fading lobes around known RGs that formed in the aftermath of earlier AGN activity episodes. Could a large fraction of sufficiently old RGs turn out to be GRGs, once such sensitivity improvements start providing evidence of earlier and earlier AGN activity episodes? Up to now, it has been informative to include all visible plasma in the angular length measurement. In future images, some of the visible plasma might be of such low pressure that it has become physically insignificant, in the sense that it does not affect the thermodynamics of the surrounding IGM anymore; we propose to exclude such plasma from a radio galaxy length.}\]
Due to its lobes, a radio galaxy whose axis aligns closely to the line of sight has a non-vanishing projected proper length. In the left column, we show LoTSS DR2 6′ cutouts of 5′ × 5′ extent centred around hosts of likely line-of-sight RGs. In the right column, we show corresponding DESI Legacy Imaging Surveys (g, r, z)-cutouts. Top: J125027.47+642034.3. Middle: J092220.72+560234.9. Bottom: J023100.61+032922.1.

5.2. Moving beyond line segment projection

In this work, we have adopted a classical approach to treating projection, by modelling radio galaxies as line segments. When a radio galaxy’s inclination angle θ is close to 0° or 180°, this approach predicts that the angular length φ and thus the projected proper length $l_p$ vanishes. In reality however, because radio galaxy lobes have non-zero volumes, the RV $L_0$ will never tend to zero. Figure 24 illustrates this point through three sources interpreted to be radio galaxies aligned closely with the line of sight. In each case, φ remains of arcminute scale. A more realistic approach, outlined in Appendix A.3, moves beyond the simplistic line segment geometry by adding two lobes of radius $R$ to the endpoints of the line segment, whose length is given by RV $L$. The ratio $\frac{2}{R}$, which was unbounded under the classical approach, is now at most $1 + \frac{1}{\eta}$, where $\eta = \frac{2R}{L}$.

5.3. Unmodelled selection effects

In this work, we have modelled both an angular length and a surface brightness selection effect. Several other plausible selection effects have not been included in the forward model, as we have judged each to be of minor importance. However, in unison, they could have a non-negligible influence on the distribution of the observed projected proper GRG length RV $L_{p,\text{obs}} | L_{p,\text{obs}} > l_{p,\text{GRG}}$. Some of their influence might have been absorbed by the parameters of the included selection effects — i.e. by $\frac{1}{2}(\phi_{\text{max}} - \phi_{\text{min}})$, $b_{\text{ref}}$, and $\sigma_{\text{ref}}$ — or, worse still, by $\xi$. One of these unmodelled selection effects is that RGs whose axes are oriented almost parallel to the line of sight are more likely to be rejected from a sample than RGs whose axes are closer to the plane of the sky, as the former do not always have a characteristic double-lobe appearance. For instance, perhaps not all readers would regard our identification of the sources as RG convincing. Nevertheless, as remarked in Sect. 2.11, conditioning $L_{p,\text{obs}} | L_{p,\text{obs}} > l_{p,\text{GRG}}$ on inclination angle does not affect its distribution. As a result, this selection effect does not necessitate forward model modifications. Furthermore, as shown in Sect. 4.4, there is a selection effect at play that favours the selection of quasar giants over non-quasar giants at high redshift, as host galaxies with quasars are more luminous in the optical and therefore have a better chance to be picked up in optical imagery. This selection effect will get less severe once deeper photometric surveys become available.

We encountered three more selection effects during our LoTSS DR2 GRG search. The larger an RG — and especially an FRII RG — becomes, the harder it is for an observer to identify its host galaxy, as an increasing number of plausible host candidates can lie interspersed in the strip of sky between the lobes. The severity of this effect, which diminishes the prevalence of the largest GRGs in a sample, depends on the balance chosen between avoiding false discoveries and avoiding rejections of true discoveries. Another selection effect runs against RGs in galaxy clusters. Such environments can contain multiple adjacent RGs, making it at times unclear which lobe belongs to which RG. When no confident double-lobe associations can be made, the RGs involved fail to make it into the sample. If galaxy clusters contain primarily smaller GRGs, this selection effect induces a bias against smaller GRGs. A final unmodelled bias runs against RGs at the end of their life cycle. Once the AGN stops launching jets for a prolonged period, it becomes hard to identify the host galaxy, which will no longer present as a bright, compact radio source. This effect preferentially deselects larger GRGs, which are even more likely to approach the end of their life than smaller GRGs.

5.4. Does the choice of prior matter?

We have taken a conservative approach to constraining the posterior distribution through the prior: we have left tail index $\xi$ and angular length selection half-width $\frac{1}{2}(\phi_{\text{max}} - \phi_{\text{min}})$. 

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$\sigma_{\text{ref}}(\nu_{\text{obs}} = 144\text{ MHz}, \theta_{\text{FWHM}} = 6''') = 25\text{ Jy}\text{ deg}^{-2}$. 

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22 Our LoTSS DR2 GRG $\phi$ and $l_p$ correspond to $b_{\text{obs}} = 1 \cdot \sigma_{\text{obs}}$; on average, $\sigma_{\text{obs}}(\nu_{\text{obs}} = 144\text{ MHz}, \theta_{\text{FWHM}} = 6''') = 25\text{ Jy}\text{ deg}^{-2}$. 

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Article number, page 27 of 68
fully unconstrained, and have adopted wide Gaussian priors for reference surface brightness parameters \(b_{\text{ref}}\) and \(\sigma_{\text{ref}}\), despite measuring them explicitly under assumptions. We provide summary statistics of the posterior in Table 4 and visualise its one- and two-parameter marginals in Fig. 19. Does our choice of prior significantly affect the inferences? To explore this question, we choose a different reasonable prior and compare results. One such prior is the fully uniform prior, which equivalences the posterior and the likelihood function. We provide analogous summary statistics of the likelihood function in Table D.1 and visualise analogous marginals in Fig. D.1. Reassuringly, no statistically significant parameter changes occur. In particular, for \(z_{\text{max}} = 0.5\), \(\xi = -3.5 \pm 0.5\) becomes \(\xi = -3.4 \pm 0.5\) upon changing to the uniform prior; for \(z_{\text{max}} = 0.25\), \(\xi = -3.5 \pm 0.4\) even remains the same. However, given the strong likelihood degeneracy between \(\xi\) and \(b_{\text{ref}}\), apparent in Fig. D.1, more stringent priors on \(b_{\text{ref}}\) are able to meaningfully shift \(\xi\)’s posterior mean. Such priors shall be appropriate only after studying the surface brightness properties of large radio galaxies — and the associated selection effect — in more detail.

5.5. Cosmological evolution of the GRG length distribution

In this work we have assumed that the parameter \(\xi\), which fully characterises the intrinsic GRG length distribution under the ansatz of Paretoinity, remains constant throughout the chosen redshift range \([0, z_{\text{max}}]\). To test this assumption, we have analysed our LoTSS DR2 GRG sample in Sect. 4 up to both \(z_{\text{max}} = 0.5\) and \(z_{\text{max}} = 0.25\). For \(z_{\text{max}} = 0.5\), we included giants that existed in the last 5 Gyr of the Universe’s history, and found \(\xi = -3.5 \pm 0.5\). Meanwhile, for \(z_{\text{max}} = 0.25\), we included giants that existed in the last 3 Gyr of the Universe’s history only, and found the very similar \(\xi = -3.5 \pm 0.4\). Thus, our analysis does not produce evidence that \(\xi\) evolves over cosmic time. However, given the large error bars, a modest time evolution cannot be excluded. Furthermore, the data sets that underlie these inferences are not disjoint: the 800 GRGs that inform the lower-maximum-redshift analysis make up 55% of the 1462 GRGs that inform the higher-maximum-redshift analysis. Whether a time evolution of \(\xi\) is expected is presumably tied to whether giant radio galaxy growth varies with environmental density at a given epoch, because the combined effects of the Universe’s expansion and ongoing large-scale structure formation can similarly change environmental density. Combining our LoTSS DR2 GRG sample with Cosmic Web reconstructions to explore giant growth as a function of environmental density is the topic of a forthcoming work (Oei et al. in prep.c). Interestingly, the recent exploration by Lan & Prochaska (2021) compared the environments of giants and non-giants did not find significant differences. The most straightforward model extension is to again assume that RG lengths are Pareto distributed with tail index \(\xi\), but now \(\xi = \xi(z)\). This function’s first-degree Maclaurin polynomial, which provides the linearisation at the present day, is

\[
\xi(z) \approx \xi(z = 0) + \frac{d\xi}{dz}(z = 0) \cdot z. \tag{42}
\]

One would adopt \(\xi(z = 0)\) and \(\frac{d\xi}{dz}(z = 0)\) as the parameters of interest, replacing what used to be a constant \(\xi\); the number of model parameters would thus increase by one. However, an attempt to infer the cosmic evolution of \(\xi\) appears promising only once the major selection effects are better constrained.

6. Conclusions

In this work, we have performed Bayesian inference on a LoTSS DR2–derived sample of 2050 giant radio galaxy projected proper lengths, using a one-parameter model that assumes a spatially homogeneous, non-evolving population of radio galaxies with stick-like geometry, Pareto-distributed lengths and isotropic inclination angles. Before fitting to data, we extend the forward model with two selection effects typical of contemporary manual GRG search campaigns. The best-fit survival function tightly reproduces the empirical one, leaving permille-scale absolute residuals. Having quantified the most important selection effects, we estimate the true comoving giant radio galaxy number density in the Local Universe.

1. We develop an analytical model through which statistical questions about radio galaxy (RG) lengths can be rigorously answered. In the current work, we apply this model to giant radio galaxies. We adopt the ansatz that the RG intrinsic proper length \(L\) as measured in three spatial dimensions, is a random variable \((RV)\) with a Pareto Type I distribution (i.e. a simple power-law distribution) characterised by tail index \(\xi\). Next, by assuming that RGs have no preferential orientation with respect to the observer, we derive the distribution of the RG projected proper length \(L_p\). By conditioning, one obtains the version relevant for GRGs, \(L_p \mid L_p > L_p,\text{GRG}\) (where we choose \(L_p,\text{GRG} = 0.7\) Mpc). This RV is again Paretian, with the same tail index \(\xi\). In summary, for giant radio galaxies, projection retains Paretianity. Finally, observers face selection effects; we model the observed projected proper length \(L_{p,\text{obs}}\) by considering an angular length threshold selection effect and a surface brightness selection effect. The angular length threshold selection effect assumes a linearly increasing angular-length-dependent probability of sample inclusion around a particular pre-defined threshold, meant to emulate manual visual searches that only target RGs of some angular length and above. The surface brightness selection effect assumes that GRGs are self-similar, and that their lobes have lognormally distributed surface brightnesses which must be above-noise to secure sample inclusion. The \(\text{GRG observed }L_{p,\text{obs}} \mid L_{p,\text{obs}} > L_{p,\text{GRG}}\) again follows through conditioning. We assume our data to be realisations of this RV.

2. The model also yields explicit expressions for the (posterior) distribution of \(L \mid L_p = L_p\). This allows one to deproject RGs in a statistical sense, providing the intrinsic proper length given the projected proper length in the limit of negligible selection effects. We also present practical expressions for the mean and variance of \(L \mid L_p = L_p\). To unravel the driving factors that allow some RGs to become GRGs, most authors search for correlations between host or environmental physical parameters and the GRG projected proper length. However, if there is a causal chain that connects host or environmental parameters to GRG length, the connection will be to the intrinsic proper length; the observer’s vantage point does not play a role in the physics. Therefore, the projection effect merely serves as a multiplicative noise source. We suggest that future analyses should recognise the projection effect as such, and correlate host or environmental parameters with the intrinsic, rather than projected, proper length — using statistical deprojection.

3. Through a manual visual search of the LoTSS DR2, the latest version of the LOFAR’s Northern Sky survey at 144 MHz, we discover a population of 2050 previously unknown gi-
The largest radio galaxy length distribution with the LoTSS DR2

Acknowledgements.

M.S.S.L. Oei et al.: Measuring the giant radio galaxy length distribution with the LoTSS DR2

Acknowledgements.

This is the largest single contribution to the literature yet, and more than doubles the community-wide consensus. We present 11 discoveries with $l_p \geq 4$ Mpc, 53 with $3 \leq l_p < 4$ Mpc, 289 with $2 \leq l_p < 3$ Mpc, 1212 with $1 \leq l_p < 2$ Mpc, and 485 with $0.7 \leq l_p < 1$ Mpc. Our study extends the known breadth of the giant radio galaxy phenomenon. Among the findings are both the GRG hosted by J081956.4+323537.6 and Alcyoneus (Oei et al. 2022a), at $l_p = 5.1$ Mpc and $l_p = 5.0$ Mpc the projectively largest giants ever found. We discover that multi-Mpc radio galaxies can be generated before redshift 1, despite the Universe’s mean density being an order of magnitude higher, and by spiral galaxies, whose stellar masses are typically an order of magnitude lower than those of ellipticals. We discover giants whose hosts have a record-low stellar mass $M_*$ = 4.8 $10^{10} M_\odot$. We also discover giants whose hosts have a record-high supermassive black hole mass $M_\bullet \geq 5.10^{10} M_\odot$; interestingly, with $l_p = 0.8$ Mpc, one of these giants is relatively small. We more than double the number of known GRGs with angular lengths exceeding that of the Moon; one discovery, at 2.2°, is the angularly largest radio galaxy in the Northern Sky and the angularly largest giant overall.

Excitingly, our LoTSS DR2 search has been far from exhaustive: many thousands of readily identifiable giants still await discovery in this public data set.

4. Using our LoTSS DR2 GRG sample up to $z_{\text{max}} = 0.5$, we generate a posterior distribution over $\xi$ and three selection effect parameters. Our model provides an excellent fit to the data, with absolute residuals being on average 2%. We infer that the intrinsic proper length distribution of the largest radio galaxies resembles a Pareto distribution with tail index $\xi = -3.5 \pm 0.5$. Our analysis does not yield evidence for an evolving $\xi$ in the last 5 Gyr of cosmic time.

5. The selection effect parameters estimated through the posterior are far from nuisance parameters, as they allow us to statistically undo the selection effects imprinted on our LoTSS DR2 GRG data. As a result, we can for the first time estimate the true comoving giant radio galaxy number density $n_{\text{GRG}}$ in the Local Universe up to $z_{\text{max}}$. We rely on the crucial assumption that the surface brightness distribution of RGs with intrinsic length $l_{\text{int}} = 0.7$ Mpc at $z = 0$ and frequency $\nu_{\text{obs}} = 144$ MHz is unimodal — and lognormal in particular. We furthermore assume a lobe spectral index $\alpha = -1$ and self-similar growth. We find $n_{\text{GRG}}(l_p, l_p = 0.7$ Mpc, $z_{\text{max}} = 0.5) = 5 \pm 2 (100$ Mpc$)^{-3}$

The implication is that giant radio galaxies are truly rare — not only from a current observational perspective, but also from a cosmological one. Current GRG lobes occupy just a few millioths of the IGM volume. At any given moment in time, most clusters and filaments — the building blocks of modern large-scale structure — do not harbour giants.

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Appendix A: Framework derivations and details

Appendix A.1: Intrinsic proper length

Let \( f_L : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0} \) be the PDF of the distribution of intrinsic (i.e. 3D) proper (i.e. not comoving) radio galaxy lengths \( L \). We will assume that \( f_L \) follows a power law between \( l_{\text{min}} \) and \( l_{\text{max}} \):

\[
 f_L (l) = \begin{cases} 
 0 & l \leq l_{\text{min}}, \text{ or otherwise.} \\
 \frac{1}{l_{\text{max}}} \left( \frac{l}{l_{\text{max}}} \right)^{\xi} & l \geq l_{\text{min}}.
\end{cases}
\]  

(A.1)

Then, when \( \xi \neq -1 \),

\[
1 = \int_{\mathbb{R}} f_L(l) \, dl = \int_{l_{\text{min}}}^{l_{\text{max}}} f_L(l_{\text{ref}}) \left( \frac{l}{l_{\text{ref}}} \right)^{\xi} \, dl = \frac{l_{\text{max}}}{\xi + 1} \left( \frac{l_{\text{max}}}{l_{\text{ref}}} \right)^{\xi+1} - \frac{l_{\text{min}}}{\xi + 1} \left( \frac{l_{\text{min}}}{l_{\text{ref}}} \right)^{\xi+1}.
\]  

(A.2)

This equation provides the normalisation factor \( f_L(l_{\text{ref}}) \) given \( l_{\text{min}}, l_{\text{max}}, \xi \) and an arbitrary choice for \( l_{\text{ref}} \neq 0 \). When \( l_{\text{max}} = \infty \), \( L \) has a Pareto Type I distribution and we must have \( \xi < -1 \) for the integrals of Eq. A.2 to converge. Throughout the remaining analysis, for the sake of simplicity, we will assume \( l_{\text{max}} = \infty \) and \( \xi < -1 \) and choose \( l_{\text{ref}} = l_{\text{min}} \). Then \( f_L(l_{\text{ref}}) = f_L(l_{\text{min}}) = \frac{1}{l_{\text{max}}} \).

The corresponding CDF is

\[
 F_L(l) = \begin{cases} 
 0 & l \leq l_{\text{min}} \\
 1 - \left( \frac{l}{l_{\text{max}}} \right)^{\xi+1} & l > l_{\text{min}}.
\end{cases}
\]  

(A.3)

In reality, radio galaxies cannot become arbitrarily long (Hardcastle 2018), at some distance from the host, all energy carried by the jets will have been radiated away, used to perform work on the IGM, transferred to CMB photons through inverse Compton scattering, or converted into heat. However, at the moment of writing, the implied maximum length \( l_{\text{max}} \) remains ill-constrained. As we will see throughout Appendix A, a major advantage of simple model assumptions is that explicit and thus insightful analytic expressions can be derived. Such easy-to-evaluate expressions complement the results of expensive numerical simulations, which aim to maximise realism; our aim to maximise insight is best served by setting \( l_{\text{max}} = \infty \). From Eq. A.3, we see that for \( l_{\text{min}} = 0.7 \text{ Mpc} \) and \( \xi = -3.5 \), the tail of the intrinsic proper length distribution \( l > 5 \text{ Mpc} \) contains less than 1% of probability. Thus, for most framework applications, our choice \( l_{\text{max}} = \infty \) is unproblematic except when higher powers of \( L \) are involved. For example, the volume-filling fraction calculations of Appendix A.10 necessitate considering \( L^4 \); for realistic \( \xi \), results exist only for finite \( l_{\text{max}} \).

Upon relabelling \( \xi \rightarrow -\alpha - 1 \), \( l_{\text{min}} \rightarrow k \), \( L \rightarrow X \) and \( l \rightarrow x \), one obtains the literature’s most common form of the Pareto Type I PDF:

\[
 f_X (x) = \begin{cases} 
 \frac{x^{\alpha+1}}{k^{\alpha+1} \Gamma(\frac{\alpha+1}{2})} & x \geq k, \\
 0 & x < k.
\end{cases}
\]  

(A.4)

Appendix A.2: Projected proper length

Appendix A.2.1: Distribution for RGs

Let \( f_{L_p} : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0} \) be the PDF of the distribution of projected proper radio galaxy lengths. The PDF \( f_{L_p} \) follows from the associated CDF \( F_{L_p} : \mathbb{R} \rightarrow [0, 1] \) through differentiation; \( F_{L_p} \) and \( f_L \) relate through

\[
 F_{L_p} (l_p) := \mathbb{P} (L_p \leq l_p) = \int_{l_p}^{l} \mathbb{P} (L \sin \Theta \leq l_p \mid L = l) f_L (l) \, dl
 = \int_{l_p}^{l} \mathbb{P} (\sin \Theta \leq \frac{l_p}{l} \mid L = l) f_L (l) \, dl.
\]  

(A.5)

Note that \( F_{L_p} (l_p) \) vanishes for \( l_p \leq 0 \); \( f_L (l) \) vanishes when \( l \) is negative, whilst \( \mathbb{P} (\sin \Theta \leq \frac{l_p}{l} \mid L = l) \) vanishes when \( l \) is positive. Clearly, the interesting case is \( l_p > 0 \). We can differentiate between two cases: \( l_p \leq l_{\text{min}} \) and \( l_p > l_{\text{min}} \). In the first case, considering that \( f_L \) has support from \( l_{\text{min}} \) onwards only, we have \( l_p < l \leq 1 \), and

\[
 F_{L_p} (l_p) = \int_{l_{\text{min}}}^{\infty} \mathbb{P} (\Theta \leq \arcsin \frac{l_p}{l}) \left( \frac{l}{l_{\text{min}}} \right)^{\xi+1} \, dl
 = \frac{\xi + 1}{l_{\text{min}}^{\xi+1}} \int_{l_{\text{min}}}^{\infty} \mathbb{P} (\Theta \leq \arcsin \frac{l_p}{l}) \left( \frac{l}{l_{\text{min}}} \right)^{\xi} \, dl.
\]

(A.6)

In the second case, we split up the integral in two:

\[
 F_{L_p} (l_p) = \int_{l_{\text{min}}}^{l_p} \mathbb{P} (\sin \Theta \leq \frac{l_p}{l}) f_L (l) \, dl
 + \int_{l_p}^{l_{\text{max}}} \mathbb{P} (\sin \Theta \leq \frac{l_p}{l}) f_L (l) \, dl
 = \int_{l_{\text{min}}}^{l_p} f_L (l) \, dl + \int_{l_p}^{l_{\text{max}}} \left( 1 - \frac{l_p^2}{l^2} \right)^{\frac{\xi+1}{2}} f_L (l) \, dl
 = 1 - \left( \frac{l_p}{l_{\text{min}}} \right)^{\xi+1} + \left( \frac{l_p}{l_{\text{min}}} \right)^{\xi+1}
 + (\xi + 1) \int_{l_p}^{l_{\text{max}}} \frac{l}{l_{\text{min}}} \left( \frac{l_p^2}{l^2} \right)^{\frac{\xi+1}{2}} \, dl
 = 1 + (\xi + 1) \int_{\frac{l_p}{l_{\text{max}}}}^{1} \sqrt{\frac{l_p}{l_{\text{min}}}} \left( \frac{l_p}{l} \right)^{\xi+1} \Gamma \left( -\frac{\xi}{2} - \frac{1}{2} \right) \, dl.
\]

(A.7)

In summary,

\[
 F_{L_p} (l_p) = \begin{cases} 
 0 & l_p \leq 0 \\
 1 + (\xi + 1) \int_{\frac{l_p}{l_{\text{max}}}}^{1} \sqrt{\frac{l_p}{l_{\text{min}}}} \left( \frac{l_p}{l} \right)^{\xi+1} \Gamma \left( -\frac{\xi}{2} - \frac{1}{2} \right) \, dl & \text{if } 0 < l_p \leq l_{\text{min}} \\
 1 + (\xi + 1) \frac{\pi}{4} \left( \frac{l_p}{l_{\text{max}}} \right)^{\xi+1} \frac{\Gamma \left( -\frac{\xi}{2} - \frac{1}{2} \right)}{\Gamma \left( -\frac{\xi}{2} + 1 \right)} & \text{if } l_p > l_{\text{min}}.
\end{cases}
\]  

(A.8)
Through differentiation,
\[
I_{f_\ell} (\ell_p) = \begin{cases} 
0 & \text{if } \ell_p \leq 0 \\
\xi+1 \left( \frac{\ell_p}{l_{\min}} \right)^\xi & \text{if } 0 < \ell_p \leq l_{\min} \\
\frac{\xi+1}{\xi} \frac{\ell_p^{\xi+1}}{l_{\min}^{\xi+2}} & \text{if } \ell_p > l_{\min},
\end{cases}
\]
where
\[
I (a, b) := \int_a^b \frac{d\eta}{\sqrt{\eta^2 - b^2}} \text{ for } a < 0, |b| < 1.
\]

Appendix A.2.2: Distribution for GRGs

To derive the projected length distribution for GRGs, we consider the distribution of the conditioned RV \( L_p | L_p > \ell_p,GRG \):
\[
F_{L_p | L_p > \ell_p,GRG} (\ell_p) = \frac{1 - F_{\ell_p} (\ell_p)}{1 - F_{\ell_p,GRG} (\ell_p)}.
\]

For \( \ell_p > \ell_p,GRG \), this reduces to
\[
F_{L_p | L_p > \ell_p,GRG} (\ell_p) = 1 - \frac{1 - F_{\ell_p} (\ell_p)}{1 - F_{\ell_p,GRG} (\ell_p)}.
\]
Furthermore assuming \( \ell_p,GRG > l_{\min} \), we twice use the bottom expression of Eq. [A.8] to obtain the final CDF expression. As before, the corresponding PDF follows through differentiation. We find
\[
F_{L_p | L_p > \ell_p,GRG} (\ell_p) = \begin{cases} 
0 & \text{if } \ell_p \leq \ell_p,GRG \\
1 - \left( \frac{\ell_p}{\ell_p,GRG} \right)^{\xi+1} & \text{if } \ell_p > \ell_p,GRG.
\end{cases}
\]
\[
f_{L_p | L_p > \ell_p,GRG} (\ell_p) = \begin{cases} 
0 & \text{if } \ell_p \leq \ell_p,GRG \\
\frac{\xi+1}{\xi} \frac{\ell_p^{\xi+1}}{l_{\min}^{\xi+2}} & \text{if } \ell_p > \ell_p,GRG.
\end{cases}
\]

Thus, the associated survival function is
\[
\mathbb{P} (L_p > \ell_p | L_p > \ell_p,GRG) = \left( \frac{\ell_p}{\ell_p,GRG} \right)^{\xi+1}.
\]
The mean projected proper length of GRGs follows from the PDF by direct computation:
\[
\mathbb{E}[L_p | L_p > \ell_p,GRG] := \int_{-\infty}^\infty f_{L_p | L_p > \ell_p,GRG} (\ell_p) \cdot \ell_p \, d\ell_p
\]
\[
= \ell_p,GRG \frac{\xi+1}{\xi+2}.
\]

23 For \( l_{max} < \infty \), this statement does not hold exactly.

Appendix A.3: Deprojection factor

Consider a radio galaxy (RG) with a projected proper length \( l_p \). Let the inclination angle \( \theta \) denote the angle between the RG’s central axis and the line of sight. The RG’s inclination angle, projected proper length and intrinsic proper length \( l \) relate via \( l_p = l \sin \theta \). Switching to random variable (RV) notation by using capital letters, the intrinsic proper length (which is the most physically relevant quantity) follows from the projected proper length (which can be measured) and the inclination angle (which is typically unknown), according to
\[
L = \frac{1}{\sin \theta} L_p.
\]

Appendix A.3.1: Without lobes

Calling the deprojection factor \( D := (\sin \theta)^{-1} \), we now calculate the distribution of \( D \) for \( f_\theta (\theta) = \frac{1}{\pi} \sin \theta; \theta \in [0, \pi] \). The result is a continuous univariate distribution without parameters supported on the semi-infinite interval \((1, \infty) \). Let \( F_D : \mathbb{R} \to [0, 1] \) be the cumulative density function (CDF) of \( D \). Then, for \( d > 1 \),
\[
F_D (d) := \mathbb{P} (D \leq d) = \mathbb{P} \left( \sin \theta \geq \frac{1}{d} \right)
= \mathbb{P} \left( \arcsin \frac{1}{d} \leq \theta < \pi - \arcsin \frac{1}{d} \right)
= F_\theta \left( \pi - \arcsin \frac{1}{d} \right) - F_\theta \left( \arcsin \frac{1}{d} \right)
= \cos \arcsin \frac{1}{d} = \sqrt{1 - \frac{1}{d^2}}.
\]

Meanwhile, \( F_D (d) = 0 \) for \( d \leq 1 \). The quantile function \( F_D^{-1} : [0, 1] \to [1, \infty) \) follows from solving \( F_D (d) = p \) for \( d \);
\[
F_D^{-1} (p) = \frac{1}{\sqrt{1 - p^2}}.
\]

Thus, the minimum factor is \( F_D^{-1} (p = 0) = 1 \), the median \( F_D^{-1} (p = \frac{1}{2}) = \frac{\sqrt{2}}{2} \), \( F_D^{-1} (p = \frac{1}{4}) = \frac{\sqrt{2}}{2} = \sqrt{2} \), \( F_D^{-1} (p = \frac{3}{4}) = \frac{\sqrt{2}}{2} \sqrt{3} = 2 \), and factors can grow arbitrarily large: \( F_D^{-1} (p) \to \infty \) as \( p \to 1 \). We conclude that half of all RGs have an intrinsic proper length more than \( \frac{\sqrt{2}}{2} \) their projected proper length.

By differentiating \( F_D \) to \( d \) we obtain \( f_D : \mathbb{R} \to \mathbb{R}_{\geq 0} \), the probability density function (PDF) of \( D \):
\[
f_D (d) = \begin{cases} 
\frac{1}{\sqrt{1 - d^2}} & \text{if } d > 1; \\
0 & \text{if } d \leq 1.
\end{cases}
\]
The mean of \( D \) is \( \mathbb{E} (D) = \frac{\pi}{2} \); the variance of \( D \) is undefined, as the corresponding integral diverges. Because \( f_D (d) \) has no maximum, the mode is undefined too. The PDF and CDF of \( D \) are shown in the upper left and right panels of Fig. [A.3] respectively.

Appendix A.3.2: With lobes

The simplest model that includes a pair of lobes approximates them as spheres of radius \( R \), whose centres are connected by a line segment of length \( L \). Regardless of the viewing angle, the
Fig. A.1: How much longer than its projected length is an RG randomly oriented with respect to the observer? We show the PDF \( f_D \) (left) and CDF \( F_D \) (right) of the deprojection factor RV \( D \). Top row: model without lobes. Of the three canonical measures of central tendency, only the median and mean exist (and equal \( \frac{1}{\sqrt{3}} \) and \( \frac{2}{\eta} \), respectively). Bottom row: model with lobes. The distribution of \( D \) now depends on a parameter: the ratio \( \eta \) between the lobe diameter \( 2R \) and the distance between the lobe centres \( L \). The model without lobes is recovered in the limit \( \eta \rightarrow 0 \).

spheres retain their size, in contrast to the line segment connecting them. Calling \( \eta := \frac{2R}{L} \in \mathbb{R}_{>0} \), we have

\[
D := \frac{L + 2R}{L \sin \Theta + 2R} = \frac{1 + \eta}{\sin \Theta + \eta}.
\] (A.21)

Again assuming \( f_\theta (\theta) = \frac{1}{2} \sin \theta; \theta \in [0, \pi] \), we repeat the derivation and find

\[
F_D (d) = \begin{cases} 
0 & \text{if } d \leq 1; \\
\sqrt{1 - \left(1 + \eta \right)^2 - \eta^2} & \text{if } 1 < d < \frac{1}{\eta} + 1; \\
1 & \text{if } d \geq \frac{1}{\eta} + 1.
\end{cases}
\] (A.22)

\[
f_D (d) = \begin{cases} 
0 & \text{if } d \leq 1; \\
\frac{(1+\eta)(\eta-\eta \eta)}{\sqrt{2}-1} & \text{if } 1 < d < \frac{1}{\eta} + 1; \\
0 & \text{if } d \geq \frac{1}{\eta} + 1.
\end{cases}
\] (A.23)

The quantile function becomes

\[
F_D^{-1} (p) = \frac{1 + \eta}{\sqrt{1 - p^2} + \eta}.
\] (A.24)

The minimum factor remains \( F_D^{-1} (p = 0) = 1 \), but a maximum factor now exists: \( F_D^{-1} (p = 1) = \frac{1}{\eta} + 1 \). The median of \( D \) is

\[
F_D^{-1} (p = \frac{1}{2}) = \frac{1 + \eta}{\frac{1}{\eta} \sqrt{3} + \eta},
\] (A.25)

which tends to \( \frac{2}{\sqrt{3}} \) for \( \eta \rightarrow 0 \) (as before), and to 1 for \( \eta \rightarrow \infty \): projection ceases to be an appreciable effect when the lobes are much larger than the line segment connecting their centres. If \( \eta \neq 0 \), \( D \) has finite support, and thus the mean, variance and higher moments exist. The \( n \)-th non-central moment is

\[
\mathbb{E} [D^n] = (1 + \eta)^2 I_{n-2} (\eta) - \eta \left( 1 + \eta \right) I_{n-1} (\eta),
\] (A.26)

where

\[
I_n (\eta) := \int_1^{\frac{1}{\eta} + 1} \frac{d^n dd}{\sqrt{d^2 - (1 + \eta - \eta d)^2}}.
\] (A.27)

In particular,

\[
I_1 (\eta) = \frac{\pi}{2 (1 + \eta)}, \quad I_0 (\eta) = \frac{\ln (\sqrt{1 - \eta^2} + 1) - \ln \eta}{\sqrt{1 - \eta^2}},
\] (A.28)

\[
I_1 (\eta) = \frac{1}{(1 - \eta) \eta} \left( 1 - \frac{2 \eta^2 \arcsinh \sqrt{1 - \eta^2}}{\sqrt{1 - \eta^2}} \right).
\] (A.29)

The expectation value of \( D \) is

\[
\mathbb{E} [D] = (1 + \eta)^2 I_{-1} (\eta) - \eta (1 + \eta) I_0 (\eta);
\] (A.30)

and the identity \( \mathbb{V} [D] = \mathbb{E} [D^2] - \mathbb{E} [D]^2 \). The mode remains undefined. For typical values of \( \eta \), we show the PDF and CDF of \( D \) in the bottom left and right panels of Fig. A.1, respectively. Figure A.2 shows the median, mean and standard deviation of \( D \) as a function of \( \eta \).
Our next objective is to find the posterior PDF $f_{L|L_p=\ell_p}(l)$ through Bayes’ theorem:

$$f_{L|L_p=\ell_p}(l) = \frac{f_{L\ell_p=\ell_p}(l) f_{\ell_p}(\ell_p)}{f_{\ell_p}(\ell_p)}.$$  \hfill (A.31)

Our line of attack will be to first calculate the likelihood CDF $F_{L\ell_p=\ell_p}(\ell_p)$, and then through differentiation the likelihood PDF $f_{L\ell_p=\ell_p}(\ell_p)$. Of course, we always consider $l > 0$, and

$$F_{L\ell_p=\ell_p}(\ell_p) := \mathbb{P}(L_p \leq \ell_p \mid L = l) = \mathbb{P}(\sin \Theta \leq \ell_p) = \begin{cases} 0 & \text{if } \ell_p \\ 2 \mathbb{P}(\Theta \leq \arcsin \frac{\ell_p}{l}) & \text{if } 0 < \ell_p \leq l \\ 1 & \text{if } \ell_p > l \end{cases}$$  \hfill (A.32)

where we have invoked Eq.\,[A.17] and the fact that $f_{\Theta}(\Theta)$ is symmetric in $\Theta = \frac{\ell_p}{l}$ for the case $0 < \ell_p < l$. Concretising this case further yields

$$F_{L\ell_p=\ell_p}(\ell_p) = 2 F_{\Theta}(\arcsin \frac{\ell_p}{l}) = 1 - \cos \arcsin \frac{\ell_p}{l} = 1 - \sqrt{1 - \left(\frac{\ell_p}{l}\right)^2}.$$  \hfill (A.33)

Through differentiation,

$$f_{L\ell_p=\ell_p}(\ell_p) = \begin{cases} \frac{1}{\ell_p} \frac{1}{\sqrt{1 - \left(\frac{\ell_p}{l}\right)^2}} & \text{if } \ell_p \leq 0 \\ \frac{1}{\ell_p^2} \frac{\ell_p}{\sqrt{1 - \left(\frac{\ell_p}{l}\right)^2}} & \text{if } 0 < \ell_p < l \\ 0 & \text{if } \ell_p \geq l \end{cases}.$$  \hfill (A.34)

Having found the likelihood, the posterior PDF follows directly through Bayes’ theorem and the PDFs computed hitherto. In concreto, when $0 < \ell_p \leq \ell_{\text{min}}$,

$$f_{L\ell_p=\ell_p}(l) = \begin{cases} \frac{1}{\ell_{\text{min}}^2} \frac{1}{\sqrt{1 - \left(\frac{l}{\ell_{\text{min}}}\right)^2}} & \text{if } l \leq \ell_{\text{min}} \\ 0 & \text{if } l > \ell_{\text{min}} \end{cases}.$$  \hfill (A.35)

whereas for $\ell_p > \ell_{\text{min}}$,

$$f_{L\ell_p=\ell_p}(l) = \begin{cases} \frac{-\xi}{2^{\xi+1} \pi^\xi} \frac{1}{\sqrt{1 + \xi^2 l^2}} & \text{if } l \leq \ell_p \\ \frac{1}{\xi^2 l^2} & \text{if } l > \ell_p \end{cases}.$$  \hfill (A.36)

Note that in both cases, for $l \gg \ell_p$, $f_{L\ell_p=\ell_p}(l) \propto \left(\frac{l}{\ell_p}\right)^{-2}$: the posterior probability density follows a power law in $l$ with exponent $-2$.

For $\ell_p \leq \ell_{\text{min}}$, the posterior mean is

$$\mathbb{E}[L \mid L_p = \ell_p] = \ell_{\text{min}} \int \frac{f_{L}(\ell_p)}{f_{\ell_p}(\ell_p)} d\ell_p,$$  \hfill (A.37)

The posterior variance $\mathbb{V}[L \mid L_p = \ell_p]$ follows from considering the second non-central moment:

$$\mathbb{V}[L \mid L_p = \ell_p] = \mathbb{E}[L^2 \mid L_p = \ell_p] - \mathbb{E}[L \mid L_p = \ell_p]^2,$$  \hfill (A.38)

so that

$$\mathbb{V}[L \mid L_p = \ell_p] = \mathbb{E}[L^2 \mid L_p = \ell_p] = \frac{2 \ell_{\text{min}}}{\xi} \int_{-\ell_{\text{min}}}^{\ell_{\text{min}}} \frac{f_{\Theta}(\Theta)}{\sqrt{1 - \Theta^2}} d\Theta.$$  \hfill (A.39)

For $\ell_p > \ell_{\text{min}}$,

$$\mathbb{E}[L^2 \mid L_p = \ell_p] = \ell_p^2 \int_{-\ell_p}^{\ell_p} \frac{f_{\Theta}(\Theta)}{\sqrt{1 - \Theta^2}} d\Theta = \ell_p^2 \int_{-\ell_p}^{\ell_p} \frac{\Theta}{\sqrt{1 - \Theta^2}} d\Theta.$$  \hfill (A.40)

Proceeding analogously,

$$\mathbb{E}[L^2 \mid L_p = \ell_p] = \frac{2 \ell_p}{\xi + 1},$$  \hfill (A.41)

so that

$$\mathbb{V}[L \mid L_p = \ell_p] = \frac{2 \ell_p}{\xi + 1} \int_{-\ell_p}^{\ell_p} \frac{f_{\Theta}(\Theta)}{\sqrt{1 - \Theta^2}} d\Theta = \frac{2 \ell_p}{\xi + 1} \int_{-\ell_p}^{\ell_p} \frac{\Theta}{\sqrt{1 - \Theta^2}} d\Theta.$$  \hfill (A.42)
In this case, both the mean and standard deviation of $L | L_p = l_p$ are proportional to $l_p$. In the table below, we list the mean and standard deviation in multiples of $l_p$ for several values of $\xi$. Since we assume $\xi < -1$, the mean and variance are guaranteed to exist. The existence of higher-order moments is $\xi$-dependent; in concreto, the highest defined order is $[-\xi]$. We prove that $L_0$ and $D$ are not independent by contradiction. If we assume that $L_0$ and $D$ are independent, then $\mathbb{E}[L_0|D] = \mathbb{E}[L_0] \mathbb{E}[D]$, or $\mathbb{E}[L] = \mathbb{E}[L] \mathbb{E}[\sin \Theta | D]$ by the independence of $L$ and $\sin \Theta$. In other words, if $L_0$ and $D$ are independent, then $1 = \mathbb{E}[\sin \Theta] \mathbb{E}[D]$. However, $\mathbb{E}[\sin \Theta] = \frac{\pi}{2}$ and $\mathbb{E}[D] = \frac{\pi}{2}$, because $1 \neq \frac{\pi}{2}$, the initial assertion must be wrong.

**Appendix A.5: GRG inclination angle**

We derive the inclination angle distribution for GRGs. The probability that a GRG has inclination angle $\theta$ is

$$f_\theta | L_p > l_p_{\text{GRG}}(\theta) \, d\theta = \frac{\mathbb{P}(\Theta = \theta, L \sin \Theta > l_p_{\text{GRG}})}{\mathbb{P}(L > l_p_{\text{GRG}})},$$

(A.43)

where we make use of the fact that the numerator’s joint probability factorises because $L$ and $\Theta$ are independent RVs. We thus find

$$f_\theta | L_p > l_p_{\text{GRG}} = \frac{1 - F_L(l_p_{\text{GRG}} \sin \theta)}{1 - F_L(l_p_{\text{GRG}})}.$$  

(A.44)

Under the Paretian assumption for $L$, we have

$$f_\theta | L_p > l_p_{\text{GRG}} = \frac{2}{(-\xi + 1) \sqrt{\pi}} \Gamma \left(-\frac{\xi}{2} + \frac{1}{2}\right) \sin^{\xi} \theta.$$  

(A.45)

**Appendix A.6: GRG angular length**

The GRG angular length RV $\Phi | L_p > l_p_{\text{GRG}}$ relates to the GRG projected proper length RV $L_p | L_p > l_p_{\text{GRG}}$ and the comoving projected proper length $R$ as

$$\Phi | L_p > l_p_{\text{GRG}} = L_p | L_p > l_p_{\text{GRG}} \cdot \frac{1 + z(R)}{R}.$$  

(A.46)

The model predicts the distribution of GRG angular lengths in the Local Universe up to comoving distance $r_{\text{max}}$. If the GRG number density is constant in the Local Universe,

$$F_R(r) = \begin{cases} 0 & \text{if } r \leq 0, \\ \frac{3}{r_{\text{max}}}^3 & \text{if } 0 < r < r_{\text{max}}, \\ 1 & \text{if } r \geq r_{\text{max}}. \end{cases}$$  

(A.47)

$$f_R(r) = \begin{cases} \frac{3}{r_{\text{max}}}^2 & \text{if } 0 < r < r_{\text{max}}, \\ 0 & \text{if } r \geq r_{\text{max}}. \end{cases}$$  

(A.48)

Because GRG life cycles are shorter than the age of the Universe, $L_p | L_p > l_p_{\text{GRG}}$ and $R$ are independent RVs. In a non-expanding universe, $z(R) = 0$, and the distribution of $\Phi | L_p > l_p_{\text{GRG}}$ can be calculated analytically. From a well-known ratio distribution identity,

$$f_{\Phi | L_p > l_p_{\text{GRG}}}(\phi) = \int_{-r_{\text{max}}}^{r_{\text{max}}} |r| \cdot f_{R | L_p > l_p_{\text{GRG}}}(\phi r) \cdot f_R(r) \, dr = \int_{0}^{r_{\text{max}}} f_{R | L_p > l_p_{\text{GRG}}}(\phi r) \cdot 3 \left( \frac{r}{r_{\text{max}}} \right)^3 \, dr.$$  

(A.49)

The integrand is non-zero only when $\phi r \geq l_p_{\text{GRG}}$, suggesting a lower integration limit of $l_p_{\text{GRG}} / \phi$. The integral vanishes altogether when $l_p_{\text{GRG}} / \phi \geq r_{\text{max}}$. Calling $\phi_{\text{GRG}} := l_p_{\text{GRG}} / r_{\text{max}}$, we have

$$f_{\Phi | L_p > l_p_{\text{GRG}}}(\phi) = \int_{\phi_{\text{GRG}}}^{r_{\text{max}}} \left|\frac{\phi}{\phi_{\text{GRG}}}\right|^{\frac{\xi + 1}{\xi}} \left(\frac{\phi}{\phi_{\text{GRG}}}\right)^{\frac{\xi}{\xi - 1}} \left(1 - \left(\frac{\phi}{\phi_{\text{GRG}}}\right)^{\frac{\xi}{\xi - 1}}\right)^{-3} \, d\phi,$$

(A.50)

where we assume $\xi \neq -1$. This PDF depends on $\xi$, $l_p_{\text{GRG}}$, and $r_{\text{max}}$ only. The CDF follows from direct integration:

$$F_{\Phi | L_p > l_p_{\text{GRG}}}(\phi) = \begin{cases} 0 & \text{if } \phi \leq \phi_{\text{GRG}}, \\ 1 + \frac{1}{\xi + 1} \left( -3 \left( \frac{\phi}{\phi_{\text{GRG}}} \right)^{\xi + 1} - (\xi + 1) \left( \frac{\phi}{\phi_{\text{GRG}}} \right)^{-3} \right) & \text{if } \phi > \phi_{\text{GRG}}. \end{cases}$$  

(A.51)

just as the mean:

$$\mathbb{E}\left[ \Phi | L_p > l_p_{\text{GRG}} \right] = \frac{\frac{\xi + 1}{\xi}}{\xi + 4} \left( \frac{1 + \phi_{\text{GRG}}}{\xi + 2} \right),$$  

(A.52)

which exists only for $\xi < -2$. By solving

$$\frac{dF_\Phi | L_p > l_p_{\text{GRG}}}{d\phi}(\phi_{\text{mode}}) = 0,$$

(A.53)

for the GRG angular length mode $\phi_{\text{mode}}$, we find

$$\phi_{\text{mode}} = \left(\frac{4}{\xi}\right)^{\frac{1}{\xi}} \cdot \phi_{\text{GRG}}.$$  

(A.54)

There is no explicit expression for the associated median. For a general non-Euclidean universe, no simple analytic form for the GRG angular length PDF appears to exist. We find an approximation valid at low redshifts by considering the Maclaurin polynomial of degree 1 for $z(r)$, the relation between comoving distance and cosmological redshift. One finds $z(r) \approx \frac{H}{r}$. As a result, for the Local Universe, Eq. (A.46) becomes

$$\Phi | L_p > l_p_{\text{GRG}} \approx L_p | L_p > l_p_{\text{GRG}} \cdot \frac{1 + \frac{H}{r}}{R},$$

(A.55)

where we use that the Hubble distance $d_H := \frac{H}{r_0}$. From a well-known inverse distribution identity,

$$f_{\frac{1}{y}}(k) = \begin{cases} 0 & \text{if } k \leq \frac{1}{r_{\text{max}}}, \\ \frac{3}{r_{\text{max}}^2} & \text{if } k > \frac{1}{r_{\text{max}}}. \end{cases}$$  

(A.56)
so that

\[ f_{\tilde{N}}(k) = \begin{cases} 0 & \text{if } k \leq \frac{1}{\tilde{n}_{\text{max}}} + \frac{1}{\tilde{d}_g} \\ \frac{3}{r_{\text{max}}(k - \frac{1}{\tilde{d}_g})} & \text{if } k > \frac{1}{\tilde{n}_{\text{max}}} + \frac{1}{\tilde{d}_g}. \end{cases} \]  

(A.57)

Combining a well-known product distribution identity with the fact that \( L_p | L_p > l_{p,\text{GRG}} \) and \( \frac{1}{\tilde{n}_{\text{max}}} + \frac{1}{\tilde{d}_g} \) are independent RVs, we find

\[ f_{\Phi | L_p > l_{p,\text{GRG}}}(\phi) = \frac{-3(\xi + 1)\phi}{r_{\text{max}}^{\xi+1} \tilde{d}_{p,\text{GRG}}} \int \frac{\partial^2}{\partial k^2} \left( \frac{1}{k - \frac{1}{\tilde{d}_g}} \right)^{\xi+1} \, dk \]  

(A.58)

when \( \phi > \phi_{\text{GRG}} + \frac{l_{\text{GRG}}}{\tilde{d}_g} \). When \( \phi \leq \phi_{\text{GRG}} + \frac{l_{\text{GRG}}}{\tilde{d}_g} \cdot f_{\Phi | L_p > l_{p,\text{GRG}}}(\phi) = 0 \). It is easy to verify that in the Euclidean limit, \( \frac{1}{\tilde{d}_g} \to 0 \), this expression reduces to that of Eq. [A.50]

Appendix A.7: Maximum likelihood estimation of the tail index

How can we estimate \( \xi \) from a sample of \( N \) GRGs? Let \( \xi_{\text{MLE}} \) be the maximum likelihood estimate (MLE) of \( \xi \). This RV is a function of \( N \) IID RVs \( \{L_{p,1}, \ldots, L_{p,N}\} \sim L_p | L_p > l_{p,\text{GRG}} \). Define the following likelihood and log-likelihood functions:

\[ \mathcal{L}(\xi) := \prod_{i=1}^{N} f_{L_p | L_p > l_{p,\text{GRG}}}(L_{p,i}) \]  

(A.59)

\[ \hat{\mathcal{L}}(\xi) := \ln(\mathcal{L}(\xi)) = N \ln(\xi) - \frac{N}{\xi + 1} \sum_{i=1}^{N} \left( \frac{L_{p,i}}{l_{p,\text{GRG}}} \right). \]  

(A.60)

Note the necessity to include a factor \( l_{p,\text{GRG}}^N \) in the definition of the log-likelihood to avoid a dimensionality error. The second derivative of \( \hat{\mathcal{L}} \) to \( \xi \) is

\[ \frac{\partial^2 \hat{\mathcal{L}}}{\partial \xi^2} = -\frac{N}{(\xi + 1)^2} < 0. \]  

(A.61)

Thus, if there exists a solution to the equation \( \frac{\partial \hat{\mathcal{L}}}{\partial \xi} = 0 \), it must correspond to a global maximum of the likelihood and log-likelihood functions:

\[ \frac{\partial \hat{\mathcal{L}}}{\partial \xi} (\xi = \xi_{\text{MLE}}) = 0, \text{ or } \xi_{\text{MLE}} = -\frac{N}{\sum_{i=1}^{N} \ln \left( \frac{L_{p,i}}{l_{p,\text{GRG}}} \right) - 1}. \]  

(A.62)

Appendix A.8: Observed projected proper length

Appendix A.8.1: General considerations

The model can be extended to incorporate observational selection effects. The relevant effects to consider vary from RG search campaign to RG search campaign, although some formulae apply in all cases. We derive these here.

To keep our extensions simple, we assume that the survey sensitivity is sufficient to detect RGs up to some redshift \( z_{\text{max}} \) only. In addition we assume that the projected proper length distribution does not evolve between \( z = z_{\text{max}} \) and \( z = 0 \). We let \( p_{\text{obs}}(l_p, z) \) denote the probability that an RG of projected proper length \( l_p \) at cosmological redshift \( z \) is detected during the campaign. Also, \( r \) is the radial comoving distance and \( n \) is the total RG number density, counting the intrinsic number of RGs (irrespective of length) per unit of comoving volume. The observed number of RGs with projected proper length between \( l_p \) and \( l_p + dl_p \) throughout a survey covering a solid angle \( \Omega \) is

\[ dN_{p,\text{obs}}(l_p, \Omega) = \frac{\Omega}{4\pi} \int_0^{\infty} n \cdot f_{l_p}(l_p) dl_p \cdot p_{\text{obs}}(l_p, z) 4\pi r^2(z) \frac{dr}{dz}. \]  

(A.63)

The total number of RGs with projected proper length between \( l_p \) and \( l_p + dl_p \) throughout a survey with solid angle \( \Omega \) is \( dN_{l_p}(l_p, \Omega) \), where

\[ dN_{l_p}(l_p, \Omega) = \frac{\Omega}{4\pi} \int_0^{z_{\text{max}}} n \cdot f_{l_p}(l_p) dl_p \cdot 4\pi r^2(z) \frac{dr}{dz} \, dz. \]  

(A.64)

We define the completeness \( C(l_p, z_{\text{max}}) \) to be

\[ C(l_p, z_{\text{max}}) := \frac{dN_{l_p,\text{obs}}(l_p, \Omega)}{dN_{l_p}(l_p, \Omega)} = \int_0^{z_{\text{max}}} p_{\text{obs}}(l_p, z) 4\pi r^2(z) E^{-1}(z) \, dz \int_0^{z_{\text{max}}} 4\pi r^2(z) E^{-1}(z) \, dz. \]  

(A.65)

The completeness only depends on the function \( p_{\text{obs}}(l_p, z) \) and our choice of \( z_{\text{max}} \).

Let the RV \( O \) denote whether an RG picked at random within \( z < z_{\text{max}} \) is detected during the search campaign. We have \( O \sim \text{Bernoulli}(C(l_p, z_{\text{max}})) \); the parameter that determines the distribution of \( O \) is itself an RV. It immediately follows that

\[ P(O = 1 | L_p = l_p) = C(l_p, z_{\text{max}}). \]

If \( L_p \) would be discrete,

\[ P(L_p = l_p | O = 1) = \frac{P(O = 1, L_p = l_p)}{P(O = 1)} = \frac{P(O = 1 | L_p = l_p) = P(L_p = l_p) \text{ } \sum_{l_p} P(O = 1 | L_p = l_p) P(L_p = l_p).} \]  

(A.66)

Let \( L_{p,\text{obs}} \) be the observed projected proper length RV; i.e., \( L_{p,\text{obs}} := L_p | O = 1 \). Its PDF is given by the continuous analogon of the preceding equation:

\[ f_{l_p,\text{obs}}(l_p) = \int_0^{l_p} C(l_p, z_{\text{max}}) f_{l_p}(l_p) \, dr. \]  

(A.67)

Obviously, \( F_{l_p,\text{obs}}(l_p) = \int_0^{l_p} f_{l_p,\text{obs}}(r_p) \, dr_p \). The CDF \( F_{l_p,\text{obs}}(l_p) | L_{p,\text{GRG}} = l_{p,\text{GRG}} \) follows from an analogon of Eq. [A.12].

We note that multiplying \( p_{\text{obs}}(l_p, z) \) with \( l_p \) and \( z \) independent factor changes \( C(l_p, z_{\text{max}}) \) by the same factor, but leaves \( f_{l_p,\text{obs}} \) unaltered.

Appendix A.8.2: Fuzzy angular length threshold

Here we illustrate a simple extension. When performing visual searches for GRG candidates, a natural criterion is to only inspect sources with an angular length that is larger than some threshold. Researchers determine the threshold based on the amount of time available to them: lower thresholds will lead to more complete samples, but will take more time to collect.
For humans, it is hard to estimate a source’s angular length precisely by eye; as a result, some of the GRG candidates included in the project’s GRG candidate catalogue will be sources with an angular length below the threshold, whilst others will be sources with an angular length above the threshold. We idealise this situation by asserting that sources with angular length \( \phi_{\text{min}} \) or below are included in the catalogue with probability 0 (i.e. never), and that sources with an angular length \( \phi_{\text{max}} \) or above are included in the catalogue with probability 1 (i.e. always). We assume a linear increase in probability as a function of \( \phi \) for intermediate angular lengths: a source with angular length \( \phi_{\text{min}} < \phi < \phi_{\text{max}} \) is included in the catalogue with probability

\[
p_{\text{obs}}(l_p, z) = \min\left(\max\left(\frac{\phi(l_p, z) - \phi_{\text{min}}}{\phi_{\text{max}} - \phi_{\text{min}}}, 0\right), 1\right). \tag{A.68}
\]

In a flat Friedmann–Lemaître–Robertson–Walker (FLRW) universe, the angular length of an RG with projected proper length \( l_p \) at cosmological redshift \( z \) is

\[
\phi(l_p, z) = \frac{l_p (1 + z)}{r(z)}. \tag{A.69}
\]

Appendix A.8.3: Surface brightness limitations

Fanaroff–Riley class II If the surface brightness \( B_{s} \) of the lobes is proportional to some negative power \( \zeta \) of the GRG’s proper length \( L \), i.e.

\[
B_{s} = b_{s, \text{ref}} \left( \frac{L}{l_{\text{ref}}} \right)^{\zeta}, \tag{A.70}
\]

then \( B_{s} \) is Pareto distributed — just like \( L \):

\[
F_{B_{s}}(t) = \begin{cases} 
0 & \text{if } t \leq 0 \\
1 - \left( \frac{b_{s, \text{ref}}}{t} \right)^{\zeta} & \text{if } 0 < t < b_{s, \text{ref}} \\
1 & \text{if } t \geq b_{s, \text{ref}}.
\end{cases} \tag{A.71}
\]

\[
f_{B_{s}}(t) = \begin{cases} 
\frac{\zeta + 1}{\zeta} \left( \frac{b_{s, \text{ref}}}{t} \right)^{\zeta + 1} & \text{if } 0 < t < b_{s, \text{ref}} \\
0 & \text{if } t \geq b_{s, \text{ref}}.
\end{cases} \tag{A.72}
\]

If RGs are self-similar in shape, then their lobe volumes are proportional to \( L^3 \). RGs appear to retain constant lobe luminosity density over most of their lifetime, so that their lobe monochromatic emission coefficients (Rybicki & Lightman 1986) are proportional to \( L^{-3} \). As line-of-sight lengths through the lobes are proportional to \( L \), surface brightness is proportional to \( L^{-2} \). These arguments thus suggest \( \zeta = -2 \). It is a poor approximation to assume that all GRGs of proper length \( l_{\text{ref}} \) have the same surface brightness \( b_{s, \text{ref}} \); observations suggest a variability of several orders of magnitude. A better description is

\[
B_{s} = b_{s, \text{ref}} \left( \frac{L}{l_{\text{ref}}} \right)^{\zeta} S, \tag{A.73}
\]

where \( S \) is a lognormal RV whose median is 1. The PDF of \( S \) is then determined by parameter \( \sigma_{s, \text{ref}} \):

\[
f_{S}(s) = \frac{1}{\sqrt{2\pi}\sigma_{s, \text{ref}}S} \exp\left(\frac{-\ln^2 s}{2\sigma^2_{s, \text{ref}}}\right). \tag{A.74}
\]

The surface brightness CDF and PDF now become

\[
F_{B_{s}}(t) = \begin{cases} 
0 & \text{if } t \leq 0 \\
F_{S}\left(\frac{b_{s, \text{ref}}}{t} + \left(\frac{b_{s, \text{ref}}}{t}\right)^{\zeta} \int_{\frac{l_{\text{ref}}}{t}}^{\infty} s^{-\zeta} f_{S}(s) \, ds \right) & \text{if } t > 0,
\end{cases} \tag{A.75}
\]

\[
f_{B_{s}}(t) = \begin{cases} 
0 & \text{if } t \leq 0 \\
\frac{\zeta + 1}{\zeta} \left( \frac{b_{s, \text{ref}}}{t} \right)^{\zeta + 1} \int_{\frac{l_{\text{ref}}}{t}}^{\infty} s^{-\zeta} f_{S}(s) \, ds & \text{if } t > 0.
\end{cases} \tag{A.76}
\]

Note that \( B_{s} \) is not exactly Pareto distributed anymore. In a relativistic rather than Euclidean universe, surface brightness is not constant with distance. To describe RGs beyond \( z = 0 \), we introduce a final model refinement:

\[
B_{s} = \frac{b_{s, \text{ref}} \cdot S}{(1 + Z)^{3-\alpha}} \left( \frac{L}{l_{\text{ref}}} \right)^{\zeta}, \tag{A.76}
\]

where the RV \( Z \) denotes cosmological redshift and \( \alpha \) is the spectral index of the lobes. We interpret \( b_{s, \text{ref}} \cdot S \) as the (lognormally distributed) lobe surface brightness for RGs of intrinsic proper length \( l_{\text{ref}} \) at \( z = 0 \). In this case, the CDF and PDF of \( B_{s} \) are most easily determined through sampling. To sample \( Z \), we can first sample the comoving distance RV \( \tilde{R} \) instead, and subsequently use \( Z = z_{\text{rel}}(R) \). We stress that this conversion depends on cosmological parameters \( \mathfrak{M} \).

To forward model a survey’s surface brightness selection effect, we must compute

\[
p_{\text{obs}}(l_p, z) = P\left(B_{s} > b_{v, \text{th}}, l_p = l_p, Z = z \middle| B_{v, \text{th}}\right)
\]

\[
= 1 - F_{B_{s}}(l_p = l_p, Z = z; (b_{v, \text{th}})), \tag{A.77}
\]

where \( b_{v, \text{th}} > 0 \) is the surface brightness threshold. Typically, \( b_{v, \text{th}} \) is comparable to the survey’s RMS noise. What is \( F_{B_{s}}(l_p = l_p, Z = z; (b_{v, \text{th}})) \)? In the simplest case, devoid of \( S \)- and \( Z \)-dependence,

\[
F_{B_{s}}(l_p = l_p, Z = z; \Theta, \tilde{R}; (b_{v, \text{th}})) = P\left(b_{v, \text{ref}} \left( \frac{L_p}{\sin \Theta \cdot l_{\text{ref}}} \right)^{\zeta} \leq b \middle| L_p = l_p \right)
\]

\[
= P\left( \sin^{\zeta} \Theta \leq \frac{b}{b_{v, \text{ref}}} \left( \frac{L_p}{l_{\text{ref}}} \right)^{\zeta} \right)
\]

\[
= P\left( \sin \Theta \leq \tilde{s}^{\frac{1}{\zeta}} \right)
\]

\[
= \begin{cases} 
0 & \text{if } 0 \leq \tilde{s}^{\frac{1}{\zeta} \cdot \Theta} \\
2F_{\tilde{R}} \left( \arcsin \left( \tilde{s}^{\frac{1}{\zeta}} \right) \right) & \text{if } 0 < \tilde{s}^{\frac{1}{\zeta} \cdot \Theta} < b_{v, \text{ref}} \left( \frac{l_p}{l_{\text{ref}}} \right)^{\zeta} \\
1 & \text{if } b \geq b_{v, \text{ref}} \left( \frac{l_p}{l_{\text{ref}}} \right)^{\zeta}
\end{cases}
\]

\[
= 1 - \sqrt{1 - \min\left( \tilde{s}^{\frac{1}{\zeta}} \left( \max\{b, 0\} \right), 1 \right)}. \tag{A.78}
\]

where

\[
\tilde{s} = \tilde{s}(b) = \frac{b}{b_{v, \text{ref}}} \left( \frac{l_p}{l_{\text{ref}}} \right)^{\zeta}. \tag{A.79}
\]

Therefore,

\[
p_{\text{obs}}(l_p, z) = \sqrt{1 - \min\left( \tilde{s}^{\frac{1}{\zeta}} \left( b_{v, \text{th}} \right), 1 \right)}. \tag{A.80}
\]
In the most refined case, for \( b > 0 \),
\[
F_{b, r} \left| L_p = l_p, Z = z \right) = \begin{cases} \frac{b_{\text{ref}} \cdot S}{(1 + Z)^{a \cdot \sqrt{\sin \Theta}}}, & b \mid L_p = l_p, Z = z \\ \end{cases}
\]
where the minimum value of \( S \) for which an RG of projected length \( l_p \) at redshift \( z \) is detectable, is \( s_{\text{min}}(l_p, z) \). For brevity, we will simply write \( s_{\text{min}} \) instead. We find
\[
p_{\text{obs}}(l_p, z) = \int_{s_{\text{min}}}^{\infty} \sqrt{1 - \left( \frac{s_{\text{min}}}{s} \right)^2} f_S(s) ds.
\]
According to the evaluation of the CDF of the lognormally distributed RV \( S \), the minimum value of \( S \) where we will simply write \( s_{\text{min}} \) is \( \sin \cdot \Theta \cdot S \). We will simply write \( s_{\text{min}} \). Now the final integral approaches \( 1 \).

Let’s assume \( \zeta = -2 \). For \( \eta = 100 \), the approximation error is at most 0.005, and for \( \eta = 1000 \), the approximation error is at most 0.0005.

Fanaroff–Riley class I The simplest correction in which FRI RGs retain a well-defined notion of length assumes a linearly decreasing surface brightness, from some value \( b_v(0) \) at the core to zero at the RG’s two endpoints. For a symmetric FRI RG, the surface brightness profile along one of the jets is \( b_v : \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0} \), and depends on the projected proper distance from the core \( r_p \) as
\[
b_v(r_p) = \begin{cases} b_v(0) \left( 1 - \frac{r_p}{r_p^*} \right) & \text{if } 0 < r_p < \frac{r_p^*}{2} \\
0 & \text{if } r_p \geq \frac{r_p^*}{2}. \end{cases}
\]
Now we consider an FRI GRG, whose projected proper length \( l_p > l_p,\text{GRG} \) would only be observed in full in the absence of noise. In actual observations, this GRG is detected as a GRG if and only if
\[
b_v \left( \frac{l_p,\text{GRG}}{2} \right) > b_v, \text{th} \quad \text{or } b_v(0) \left( 1 - \frac{l_p,\text{GRG}}{l_p} \right) > b_v, \text{th}.
\]
Under our assumption of a linear surface brightness profile, the mean surface brightness along the jet axis \( b_v = b_v \left( \frac{1}{2} \right) \). As this must be half of the surface brightness at the core,
\[
b_v = b_v \left( \frac{l_p}{4} \right) = \frac{b_v(0)}{2}.
\]
Combining Eqs. (A.87) and (A.88) we find that the GRG is detected as such if
\[
b_v > \frac{b_v, \text{th}}{2 \left( 1 - \frac{l_p,\text{GRG}}{l_p} \right)}.
\]
Now regarding \( b_v \) as an RV and recognizing that it might behave exactly as in Eq. (A.76), we find that the surface brightness selection effect for FRI GRGs may be modelled as
\[
\bar{p}_{\text{obs}}(l_p, z) = \mathbb{P} \left( b_v > \frac{b_v, \text{th}}{2 \left( 1 - \frac{l_p,\text{GRG}}{l_p} \right)} \mid L_p = l_p, Z = z \right).
\]
We see that the full formulaic structure of \( \bar{p}_{\text{obs}}(l_p, z) \) is the same for FRI and FRII GRGs, except that for FRI GRGs a change
\[
b_v, \text{th} \rightarrow \frac{b_v, \text{th}}{2 \left( 1 - \frac{l_p,\text{GRG}}{l_p} \right)}.
\]
is necessary. There is no change for \( l_p = 2 l_p,\text{GRG} \).

Appendix A.9: GRG number density
A statistic of major interest is the number density of GRGs in the contemporary Universe. Let \( n_{\text{GRG}} \) be the comoving GRG number density, so that
\[
n_{\text{GRG}} = n \mathbb{P} \left( L_p > l_p, \text{GRG} \right) = n \left( 1 - F_{l_p} \left( l_p, \text{GRG} \right) \right).
\]

\[
N_{\text{GRG, obs}}(\Omega, z_{\text{max}}) = \int_{l_p, \text{GRG}} \mathbb{d}N_{l_p, \text{obs}} \left( l_p, \Omega \right).
\]
After invoking Eq. A.63 and isolating \( n \), we obtain
\[
n = \frac{4\pi}{\Omega} N_{\text{GRG,obs}} (\Omega, \zeta_{\text{max}}) \int_{l_{\text{GRG}}}^{l_{\text{GRG,obs}}} \int_{0}^{\zeta_{\text{obs}}} \int_{0}^{\zeta_{\text{max}}} \int_{-\pi/2}^{\pi/2} \frac{d\zeta \, \sin\varphi \, d\varphi \, d\theta}{l}.
\] (A.94)

Combining Eqs. A.8, A.9, A.92 and A.94 for \( l_{\text{p,GRG}} > l_{\text{min}} \), we arrive at
\[
n_{\text{GRG}} = \frac{4\pi}{\Omega} N_{\text{GRG,obs}} (\Omega, \zeta_{\text{max}}) \int_{l_{\text{GRG}}}^{l_{\text{GRG,obs}}} \int_{0}^{\zeta_{\text{obs}}} \int_{0}^{\zeta_{\text{max}}} \int_{-\pi/2}^{\pi/2} \frac{d\zeta \, \sin\varphi \, d\varphi \, d\theta}{l}.
\] (A.95)

From observations, we know \( \Omega \) and can \( \text{—} \) for a given \( \zeta_{\text{max}} \) \( \text{—} \) simply count \( N_{\text{GRG,obs}} (\Omega, \zeta_{\text{max}}) \). Moreover, we can fit \( \xi \) and the parameters that occur in \( p_{\text{GRG}}(l, z) \)(e.g., \( \frac{1}{2}(\phi_{\text{max}} - \phi_{\text{min}}) \), \( h_{\text{ref}} \), and \( \sigma_{\text{GRG}} \)) to the ECDF of \( L_{\text{p,GRG}} \). \( L_{\text{p,GRG}} \) > \( l_{\text{GRG}} \). Note that \( n_{\text{GRG}} \) does not depend on \( l_{\text{min}} \), which drops out through the division. However, \( n_{\text{GRG}} \) does depend on cosmological parameters through the relation between cosmological redshift \( z \) and radial comoving distance \( r \).

Appendix A.10: GRG lobe volume-filling fraction

Assuming self-similar growth, the combined proper volume \( V \) of an RG’s lobes and its intrinsic proper length \( l \) obey \( V \propto l^3 \). The constant of proportionality varies per RG and depends on the shape of the lobes; we treat it as an RV \( \Upsilon \propto \frac{l^3}{V} \). Then the proper VFF of GRG lobes \( \text{VFF}_{\text{GRG}}(z) = \text{VFF}_{\text{GRG}}(z = 0) \cdot (1 + z)^3 \), where
\[
\text{VFF}_{\text{GRG}}(z = 0) = n_{\text{GRG}} \cdot \mathbb{E}[V | L > l_{\text{p,GRG}}] = n_{\text{GRG}} \cdot \mathbb{E}[\frac{V}{l^3} | L > l_{\text{p,GRG}}]
\] (A.96)

Assuming that RGs grow self-similarly, so that shape does not reveal length, \( \Upsilon \) and \( L^3 \) are conditionally independent given \( l_{p > l_{\text{GRG}}} \). As a result,
\[
\text{VFF}_{\text{GRG}}(z = 0) = n_{\text{GRG}} \cdot \mathbb{E}[\Upsilon | L > l_{\text{p,GRG}}] \cdot \mathbb{E}[L^3 | L > l_{\text{p,GRG}}] = n_{\text{GRG}} \cdot \mathbb{E}[\Upsilon] \cdot \mathbb{E}[L^3] = \text{VFF}_{\text{GRG}}(z = 0).
\] (A.97)

To obtain this last line, we once more exploit self-similarity: \( \Upsilon | L > l_{\text{p,GRG}} \) \( \sim \Upsilon \). We can approximate \( \mathbb{E}[\Upsilon] \) by taking the mean of some \( \frac{V}{l} \) deduced from observations. A technical complication arises from the fact that \( \mathbb{E}[L^3 | L > l_{\text{p,GRG}}] \) does not exist for \( z \geq -4 \) under our model. This is an artefact of the Pareto assumption for \( L \), which unrealistically features support over an infinitely long part of the real line: \( l \in \mathbb{R}^+ \). This causes the expectation value integral to diverge for \( z \geq -4 \). An approximation to \( \text{VFF}_{\text{GRG}}(z = 0) \) that works for all \( z \) is the lower bound
\[
\text{VFF}_{\text{GRG}}(z = 0) > n_{\text{GRG}} \cdot \mathbb{E}[\Upsilon] \cdot \mathbb{E}[L | L > l_{\text{p,GRG}}].
\] (A.98)

which follows from Jensen’s inequality. Here
\[
\mathbb{E}[L | L > l_{\text{p,GRG}}] = \frac{\Gamma(\frac{z}{2} + 1)}{\Gamma(\frac{z}{2})} \frac{\Gamma(\frac{z}{2} + 1)}{\Gamma(\frac{z}{2} + 1)}
\] (A.99)

Alternative approximations, where we use \( \Upsilon = \frac{V}{l} \), are
\[
\text{VFF}_{\text{GRG}}(z = 0) > n_{\text{GRG}} \cdot \mathbb{E}[\Upsilon] \cdot \mathbb{E}[L | L > l_{\text{p,GRG}}] = \frac{4\pi}{\Omega} N_{\text{GRG,obs}} (\Omega, \zeta_{\text{max}}) \int_{l_{\text{GRG}}}^{l_{\text{GRG,obs}}} \int_{0}^{\zeta_{\text{obs}}} \int_{0}^{\zeta_{\text{max}}} \int_{-\pi/2}^{\pi/2} \frac{d\zeta \, \sin\varphi \, d\varphi \, d\theta}{l}.
\] (A.100)

and
\[
\text{VFF}_{\text{GRG}}(z = 0) > n_{\text{GRG}} \cdot \mathbb{E}[\Upsilon \cdot L | L > l_{\text{p,GRG}}] = \frac{4\pi}{\Omega} N_{\text{GRG,obs}} (\Omega, \zeta_{\text{max}}) \int_{l_{\text{GRG}}}^{l_{\text{GRG,obs}}} \int_{0}^{\zeta_{\text{obs}}} \int_{0}^{\zeta_{\text{max}}} \int_{-\pi/2}^{\pi/2} \frac{d\zeta \, \sin\varphi \, d\varphi \, d\theta}{l}.
\] (A.101)

where
\[
m_{l_{p}} \sim l_{p} \cdot \mathcal{L}_{\text{GRG}}(\Upsilon, \zeta_{\text{max}}) = \frac{0}{l_{p} \cdot \mathcal{L}_{\text{GRG}}(\Upsilon, \zeta_{\text{max}})}
\] (A.102)

is the median of the cubed projected proper length for GRGs. An advantage of these latter expressions is that there are more data available to estimate \( \mathbb{E}[\Upsilon] \) than there are to estimate \( \mathbb{E}[\Upsilon] \).

Appendix A.11: Unification model constraints from quasar and non-quasar giants

The quasar GRG probability \( p_{\Omega} \) is
\[
p_{\Omega} = \frac{\mathbb{P}(l_{p,\text{obs}} \geq l_{p,\text{GRG}}, \sin \Theta \leq \sin \theta_{\text{max}})}{\mathbb{P}(l_{p,\text{obs}} \geq l_{p,\text{GRG}})}
\] (A.103)

Now using Eqs. A.3 and A.8 and \( \sin \varphi(\chi) = \frac{2}{\sqrt{1 - x^2}} \) over the domain of integration,
\[
p_{\Omega} = \frac{4\pi}{(\xi + 1)} \frac{1}{\sqrt{\pi}} \sqrt{\frac{\pi}{\xi + 1}} \int_{0}^{\sin \theta_{\text{max}}} \frac{x^{-\xi} \, dx}{\sqrt{1 - x^2}}.
\] (A.104)

Appendix A.12: Extreme GRGs in a sample

An interesting feature of the model is its ability to predict the occurrence of GRGs with extreme projected proper lengths in a GRG sample of, say, size \( N \). Now consider some \( l_{p} > l_{p,\text{GRG}} \) \( \sim \) what is the probability \( p_{l_{p}} \) that an observed GRG will have a projected proper length exceeding \( l_{p} \)? Proceeding as in the derivation of Eq. A.12 we find
\[
p_{l_{p}} \left(l_{p}\right) := \mathbb{P}(l_{p,\text{obs}} > l_{p} | l_{p,\text{obs}} > l_{p,\text{GRG}}) = \frac{1 - F_{l_{p,\text{obs}}}(l_{p})}{1 - F_{l_{p,\text{obs}}}(l_{p,\text{GRG}})}.
\] (A.105)

In the absence of selection effects, \( p_{l_{p}} \left(l_{p}\right) \) is given by Eq. A.15. The number of GRGs with extreme projected proper lengths \( N_{l_{p}} \sim \text{Binom}(N, p_{l_{p}}(l_{p})). \) Interesting questions can be answered readily. For example, the probability that the sample contains at least one GRG with projected proper length \( l_{p} \) or larger, is
\[
\mathbb{P}(N_{l_{p}} \geq 1) = 1 - \mathbb{P}(N_{l_{p}} = 0) = 1 - \left(1 - p_{l_{p}}(l_{p})\right)^{N}.
\] (A.106)
Consider a radio galaxy at cosmological redshift $z$ of intrinsic proper length $l$ bounded by spherical lobes of radius $R$ and spectral index $\alpha$. If a fraction $f_j$ of the radio galaxy’s central axis lies within the lobes, then $R = \frac{1}{2} f_j \cdot l$. If a fraction $f_{L*}$ of the galaxy’s total luminosity density $L_*$ comes from the lobes, then the monochromatic emission coefficient $j_\nu = \frac{L_*}{4\pi c \nu} \frac{1}{R^2}$, where the lobe volume $V = \frac{4}{3} \pi R^3$. A formula of practical value features projected proper length $l_p$ instead of $l$. Given the approximate nature of our approach, we therefore simply assume $l \approx \mathbb{E}[D](\eta(f_j)) \cdot l_p$, with the deprojection factor expectation $\mathbb{E}[D]$ given in Eq. A.29 $\eta(f_j) = \frac{1}{1-f_j}$. The maximum surface brightness of the lobes as seen by an observer is

$$b_{\nu,\max} = \frac{3 f_{L*} \cdot L_*}{\pi^2 \cdot \mathbb{E}[D](\eta(f_j)) \cdot f_j^2 \cdot l_p^2 \cdot (1 + z)^{3-\alpha}}.$$  \hfill (C.1)

valid for the line of sight that pierces through a lobe along a diameter. The average line of sight length $d$ within a lobe is smaller than $d_{\max} = 2R$, though, and, given by

$$\langle d \rangle = \frac{1}{\frac{R^2}{2} - x^2} 2\pi x \ dx = \frac{4}{3} R.$$  \hfill (C.2)

Therefore, the mean surface brightness of the lobes as seen by an observer is

$$\langle b_{\nu} \rangle = \frac{d_{\max} \cdot b_{\nu,\max}}{2} = \frac{2}{3} b_{\nu,\max}.$$  \hfill (C.3)

**Appendix D: Likelihood function**

In Table D.1 we present maximum likelihood and likelihood mean and standard deviation estimates for the inference described in Section 4.1.4; one may compare the results to those in Table 2. In Fig. D.1 we visually summarise the likelihood function; one may compare to Fig. 19. The strong degeneracy between $\xi$ and $b_{\nu,\max}$, directly apparent from the central two-parameter marginal in the leftmost column of Fig. D.1 translates to a ridge of essentially constant likelihood that extends from $\xi \approx -4$ to $\xi \approx -2$. Compared to a non-degenerate case, this makes the maximum likelihood parameters both intrinsically less meaningful and more prone to numerical approximation errors.

**Table D.1:** Maximum likelihood estimate (MLE) and likelihood mean and standard deviation (SD) estimates of the free parameters in intrinsic GRG length distribution inference. The model assumes $\xi$ is constant for $z \in [0, z_{\max}]$. We determine the likelihood function twice: for $z_{\max} = 0.5$, using 1462 GRGs, and for $z_{\max} = 0.25$, using 800 GRGs. We caution that, in this case, the MLE parameters are a poor measure of central tendency.

$$\begin{array}{|c|c|c|}
\hline
\text{parameter} & \text{MLE} & \text{likelihood mean and SD} \\
\hline
\xi & -2.15 & -3.4 \pm 0.5 \\
\frac{1}{2}(f_{\max} - f_{\min}) & 1.7^\prime & 1.9 \pm 0.3^\prime \\
b_{\nu,\max} & 90 \text{ Jy deg}^{-2} & 660 \pm 400 \text{ Jy deg}^{-2} \\
\sigma_{\nu,\max} & 1.35 & 1.2 \pm 0.2 \\
\hline
\end{array}$$

For $z_{\max} = 0.25$:

$$\begin{array}{|c|c|c|}
\hline
\text{parameter} & \text{MLE} & \text{likelihood mean and SD} \\
\hline
\xi & -2.2 & -3.5 \pm 0.4 \\
\frac{1}{2}(f_{\max} - f_{\min}) & 2.25^\prime & 1.7 \pm 0.3^\prime \\
b_{\nu,\max} & 150 \text{ Jy deg}^{-2} & 1050 \pm 560 \text{ Jy deg}^{-2} \\
\sigma_{\nu,\max} & 1.25 & 1.4 \pm 0.4 \\
\hline
\end{array}$$

**Appendix E: New GRG properties**

We list properties of all 2050 giants discovered in our LoTSS DR2 search campaign. The column ‘rank’ denotes each giant’s projected proper length rank within this new sample, not within the total known population. The column ‘host coordinates’ contains the central right ascension and declination of the host galaxy. The columns ‘redshift’ and ‘redshift type’
provide cosmological redshift estimates, derived from spectroscopy \(s\) or photometry \(p\). The column ‘angular length’ denotes the largest great-circle distance between two LoTSS DR2–detectable GRG (end)points. These angular lengths may increase in future deeper surveys. We have not measured angular length errors on a case by case basis, but estimate them to be 0.15’. The column ‘projected proper length’ propagates both redshift and angular length uncertainty and assumes the \cite{Planck} cosmology. The columns ‘host stellar mass’ and ‘host SMBH mass’, further discussed in Section \ref{sec:masses} and Appendix \ref{app:masses} provide SDSS-derived estimates of host stellar and supermassive black hole masses. Finally, the column ‘host
quasar' indicates whether the host’s AGN has a quasar appearance: y (yes) or n (no).
In cases where only a set of candidates containing the host galaxy could be established beyond reasonable doubt, we list the properties of the lowest-redshift candidate. In this way, the provided projected proper length bounds the actual projected proper length from below. In such cases, to signify uncertainty, we mark the host name, host coordinates, and physical host properties in grey. We note that this lowest-redshift candidate is often, but not always, also the most probable host.
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<th>host coordinates</th>
<th>redshift</th>
<th>redshift type</th>
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<th>projected proper length (Mpc)</th>
<th>host stellar mass (10^12 M⊙)</th>
<th>host SMBH mass (10^9 M⊙)</th>
<th>host quasar</th>
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<td>124.981, 32.5938</td>
<td>0.749 ± 0.073</td>
<td>p</td>
<td>11.2</td>
<td>5.07 ± 0.02</td>
<td>n</td>
<td>0.4 ± 0.2</td>
<td>n</td>
</tr>
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<td>2</td>
<td>J081121.68+522410.0</td>
<td>123.5904, 52.4028</td>
<td>0.2467 ± 6.10^{-5}</td>
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<td>4.99 ± 0.04</td>
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<td>p</td>
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<td>4.80 ± 0.06</td>
<td>3 ± 0.2</td>
<td>2 ± 0.6</td>
<td>n</td>
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<td>198.9976, 26.4397</td>
<td>0.6230 ± 5.10^{-4}</td>
<td>s</td>
<td>11.0</td>
<td>4.62 ± 0.06</td>
<td>y</td>
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<td>p</td>
<td>10.6</td>
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Martijn S.S. Oei et al.: Measuring the giant radio galaxy length distribution with the LoTSS DR2.
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Martijn S.S.L. Oei et al.: Measuring the giant radio galaxy length distribution with the LoTSS DR2

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Martijn S.S.L. Oei et al.: Measuring the giant radio galaxy length distribution with the LoTSS DR2
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Martijn S.S.L. Oei et al.: Measuring the giant radio galaxy length distribution with the LoTSS DR2
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Martijn S.S.L. Oei et al.: Measuring the giant radio galaxy length distribution with the LoTSS DR2
Article number, page 67 of 68
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<th>Dec (J2000)</th>
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<th>s</th>
<th>0.72 ± 0.01</th>
<th>...</th>
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<td>2020</td>
<td>J152151.93+570635.0</td>
<td>230.4664</td>
<td>57.1097</td>
<td>0.8001</td>
<td>s</td>
<td>10.0</td>
<td>0.72 ± 0.01</td>
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<td>2021</td>
<td>J095605.24+415827.5</td>
<td>113.7719</td>
<td>41.9743</td>
<td>0.0873 ± 2 × 10⁻⁶</td>
<td>s</td>
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<td>0.72 ± 0.01</td>
<td>0.5 ± 0.0</td>
<td>0.9 ± 0.2</td>
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<td>2022</td>
<td>J160756.03+081218.7</td>
<td>226.9918</td>
<td>8.2052</td>
<td>0.0764 ± 1 × 10⁻⁷</td>
<td>s</td>
<td>8.0</td>
<td>0.72 ± 0.01</td>
<td>2.6 ± 0.2</td>
<td>1.0 ± 0.2</td>
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<td>2023</td>
<td>J153630.92+332435.0</td>
<td>268.7084</td>
<td>33.4097</td>
<td>0.1703 ± 2 × 10⁻⁷</td>
<td>s</td>
<td>4.0</td>
<td>0.72 ± 0.01</td>
<td>4.6 ± 0.2</td>
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<td>2024</td>
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<td>37.4121</td>
<td>28.9876</td>
<td>0.047</td>
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<td>12.6</td>
<td>0.72 ± 0.53</td>
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<td>0.0818</td>
<td>s</td>
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<td>0.72 ± 0.01</td>
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<td>27.5299</td>
<td>0.1267</td>
<td>s</td>
<td>5.1</td>
<td>0.72 ± 0.02</td>
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<td>2027</td>
<td>J101922.49+320922.0</td>
<td>154.8433</td>
<td>32.1561</td>
<td>0.0958 ± 3 × 10⁻⁷</td>
<td>s</td>
<td>6.5</td>
<td>0.72 ± 0.02</td>
<td>1.2 ± 0.1</td>
<td>0.5 ± 0.1</td>
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<td>2028</td>
<td>J084407.72+551502.0</td>
<td>131.0322</td>
<td>55.8506</td>
<td>0.2695 ± 6 × 10⁻⁷</td>
<td>s</td>
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<td>0.72 ± 0.04</td>
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<td>1.2 ± 0.4</td>
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<td>2029</td>
<td>J153010.27+290030.2</td>
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<td>29.0084</td>
<td>0.0842 ± 2 × 10⁻⁷</td>
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<td>0.72 ± 0.01</td>
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<td>1.5 ± 0.3</td>
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<td>140.3421</td>
<td>54.8650</td>
<td>0.0447 ± 1 × 10⁻⁷</td>
<td>s</td>
<td>13.1</td>
<td>0.71 ± 0.01</td>
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<td>0.9 ± 0.1</td>
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<td>2031</td>
<td>J094907.37+385110.9</td>
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<td>38.8530</td>
<td>0.1639 ± 2 × 10⁻⁷</td>
<td>s</td>
<td>4.1</td>
<td>0.71 ± 0.03</td>
<td>2.0 ± 0.1</td>
<td>0.5 ± 0.1</td>
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<td>142.3959</td>
<td>62.9498</td>
<td>0.1266 ± 2 × 10⁻⁷</td>
<td>s</td>
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<td>0.71 ± 0.02</td>
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<td>0.71 ± 0.02</td>
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<td>53.7300</td>
<td>0.5495 ± 0.0001</td>
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<td>1.8</td>
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<td>1.9 ± 1.8</td>
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<td>132.5038</td>
<td>38.5964</td>
<td>0.142 ± 0.001</td>
<td>p</td>
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<td>28.2174</td>
<td>25.5794</td>
<td>0.2137 ± 3 × 10⁻⁷</td>
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<td>26.7573</td>
<td>0.0963 ± 2 × 10⁻⁷</td>
<td>s</td>
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<td>0.1012 ± 2 × 10⁻⁷</td>
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<td>25.2094</td>
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<td>s</td>
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<td>0.70 ± 0.03</td>
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